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Informality: Family Ties and Retirement Income

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Abstract

This paper suggests a demand side analysis of informal employment characterised by incompliances with labour tax regulation, using a general equilibrium model with overlapping generations. A public social insurance provide benefits to formal employees in retirement, while we allow for an informal insurance mechanism for informal employees through a social norm of mutual support. The objective of the paper is to evaluate impact of auditing policy and social norms on growth and social welfare. We define the private transfer rate to the informal population and provide an analysis on the impact of social norms to wage levels, growth and welfare.

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1 Introduction

The second half of the twentieth century is marked by the expansion of public insurance systems, nevertheless, an important share of employees are still working informally, uncovered by public social insurance system because of the lack of social security contributions by both the employer and the worker. According to ILO 2018 report, informal employment constitutes 61.2% of the world's employed population (around two billion workers), 93% of which live in emerging and developing countries. Furthermore, contrary to the anticipations of most scholars (Beneria, 2001; Castells and Portes, 1989), the share of informal employment is rising in these economics. Informal workers face

welfare losses due to health and income concerns when they are working as well as during retirement. In many cases, poverty and informal employment goes hand in hand. An overall inegalitarian structure emerges due to the inevitable necessity to provide emergency relief for those in severe deprivation because of informality (Bugra and Keyder, 2006; Yakut Cakar, 2007; Elveren, 2008). Social norms and customs indirectly alleviate financial concerns in retirement through familial bonds. In many developing countries, they allow different support networks to deal with different risks to the cost of rationalising informal employment without much social and political controversy. Thus, in those countries, informal employment is also characterised by its persistence.

Connections of the institutional framework with informal employment is discussed in many theoretical and empirical works[1]. Analysis of interactions between social and public welfare regimes and the economy at large provides some interesting insights. For instance, Fanti and Gori (2012) compares the steady-state and dynamic outcomes of voluntary intra-family transfers and pay-as-you-go public pensions and shows that the shift from a private system to public pensions increases the gross domestic product (GDP) per worker and considerably reduces the possibility of cyclical instability. In contrast, Ulysseay (2018) suggests that informal employment has several negative economic implications such as reducing the provision of public goods as a result of tax avoidance, distorting firms' decisions on their size and misallocation of resources by enabling less productive informal firms with more productive formal firms and positive implications for growth such as providing de facto flexibility (Meghir et al., 2015).

A close up view of informal employment is articulated as a choice problem. Some scholar argue from firms' perspective, that by avoiding taxes and regulations, higher profit margins can be secured[2]. Some other scholars describe the choice of informality as a survival strategy not only by firms but also by workers[3]. Thus, in the economy, there may be informal workers because some firms are choosing to be informal to earn more profit or to escape entry regulations despite being productive enough or because some workers are low skilled and informality ensures survival in the labour market

[1]Previous literature proposed segmented labour market hypothesis (Lewis (1954)), however, empirical studies can not find strong evidence to conclude for such a segmentation in many cases. See Magnac (1991) for Colombia, Maloney (2004) for Mexico, Pratap and Quintin (2006) for Argentina and for a set of developing countries.

[2]See Farrell (2004) and Levy (2008), among others.

[3]See De Soto (1989) for the pioneering study using this approach.

by increasing employment opportunities in the labour market.

Recent studies combine above points to point out to supply and demand side factors and relax the dualistic view of the labour market. They suggest that informal employment may not be as bad as it is argued and non-pecuniary rewards and alternative employment ethics and insurance networks that they allow should also be considered to understand its persistence in developing countries (De Soto (1989), Maloney (2004), Kucera and Roconlato (2008)). Joubert (2015) provides evidence that the informal sector is not merely a residual labor market using a dynamic behavioural model under the rules of Chile's canonical privatized pension system and captures that work experience is highly transferrable from one sector to the other. From a demand side perspective, arguments such as unionisation levels, labour market rigidities and distortions created by regulations and taxes have been investigated (Galli and Kucera (2004), Heckman and Pagés-Serra (2000), Saavedra and Torero (2004)). Amaral and Quintin (2006) shows in a general equilibrium set-up that large firms operate formally and small firms informally and a competitive labour market equalises formal and informal wages for workers with the same ability. Ceni (2017) has analysed the incidence of informality and the transition between states using Argentinean panel data from 1995 to 2008 and found evidence that low-educated workers have difficulties to obtain a pension by the age of 65 and even by age 70 and shown that if the parameters are fixed as in the pay-as-you-go (PAYG) model, there is a slight reduction in formality and pension income, with stricter pension requirements, there is an increase in formality without achieving the benchmark level of pension coverage. We can also refer to dynamic models analysing the evolution of informal employment provided by Araujo and De Souza (2010) which links the impact of excessive regulatory system with dynamics of workers and firms' entrance and withdrawal of formal and informal economy and evaluates the optimal relation between regulatory and enforcement action by the government and Ozturk Goktuna and Dayangac (2011) which provides an evolutionary explanation for the existence of informal employment share in the economy given the impact of social support networks and institutional factors.

In this study, we suggest a demand side analysis characterised by incompliances with labour tax regulations. Specifically, firms avoid payroll taxes by employing informal workers at the cost of a penalty if this practice is caught in random audits of the public social security institution. In parallel, the socio-economic framework is assumed to incorporate strong social ties that

alleviate income risks in retirement. While public insurance provide benefits to formal employees in retirement, an informal insurance mechanism, through a social norm of mutual support ensures income to informal employees. We compute the level of transfers to the uncovered share of the population at the social optimum and provide an analysis of the impact of social support norm to wage levels, growth and welfare and show that at the equilibrium, the informal (alternative) insurance network reduces income risks of informal employees in retirement. This result propose a framework to account for competing insurance mechanisms alongside public insurance when the economy has a structural and persistent informality problem and gives insights on institutional factors affecting the inefficiency of policies against informality in line with similar studies family ties for economic outcomes (Alesina and Giuliano, 2013; Bau, 2021).

The paper is organised as follows: the first section describes the theoretical model, the second section provides with equilibrium and steady state of model economy and analyses impact of political and institutional factors on growth and welfare and the last section presents the conclusion.

2 The Model

The economy consists of firms, households and a public social security institution. Firms produce one good that can be used as a consumption and capital good. Households own capital and inelastically supply labour. Firms use capital and labour as inputs and could choose illegally to hire unregistered workers i.e. employ workers without paying their income taxes and pension contributions. An auditing agency operates under the public social security institution and engages in costly auditing activity to catch unregistered employment practice and a penalty is imposed to employers working unregistered labour. A part (λ) of total labour is not registered for cost minimisation purposes. Registered (formal) and unregistered (informal) workers are perfect substitutes.

2.1 Firms

Production technology has constant returns to scale and is represented by a Cobb-Douglas production function as follows:

$$y(k, l^{1,d}, l^{2,d}) = k^{\alpha}(l^{1,d} + l^{2,d})^{1-\alpha} \tag{1}$$

where $l^{1,d}$ and $l^{2,d}$ are the demands of unregistered and registered labour respectively, α is the output elasticity of capital. As firms decide on the share of different types of labour, total labour demand is expressed as follows: $l^d = l^{1,d} + l^{2,d} = \lambda l^d + (1-\lambda) l^d$. We suppose that capital depreciates fully.

Let r be the real interest rate, the total cost of one unit of capital is then $(1+r)$. The cost of registered workers includes, in addition to gross wage w, a payroll tax at a rate τ_w. The cost of employing unregistered worker equals the net wage of a registered worker (out of firms' pension contribution at a rate θ_u) and if this practice is caught through auditing, the payroll tax and a proportional penalty ϕ over the wage rate. As auditing is costly, government audits firms with probability z and the detection probability is then $f(\lambda_t) = z\lambda_t$. This can allow for a certain share of evasive behaviour in the economy. Remember that firms choose how much unregistered labour they will employ since there are no productivity differences between these workers and there might be cost minimisation in employing workers without registering them.

There is one representative firm and its objective is to maximise the profit Π with respect to labour, capital and evasion from payroll tax:

$$\underset{\{k_t, l_t^d, \lambda_t\}}{Max}\ \Pi_t = y_t - (r_t+1)k_t - (1+\tau_w)w_t l_t^{2,d} - (1-f)(1-\theta_u)w_t l_t^{1,d} - f(1+\phi+\tau_w)w_t l_t^{1,d} \tag{2}$$

Given pension contribution, input prices and penalty rule, the firm determines its optimal factor demands and as well as the share of informal employment by maximising profit.

2.2 Households

There are two periods in the lifetime of households: working and retirement. We denote the registration status in the working period by $h \in \{1, 2\}$ where $h = 1$ refers to unregistered status and $(h = 2)$ to registered. Registered workers are either registered initially or they are unregistered initially and registered after being caught in a random inspection.

In working period, households supply inelastically one unit of labour given w_t and their registration status h, invest in capital market s_t^h given r_t. In addition to private savings, we suppose that working generation share their wage at a rate θ_q with uncovered retirees and given the registration status, social security contribution is paid. During retirement period, registered

workers receive pension benefits. Retirees who are uncovered, receive transfers from working generation. This modelisation allow for a non-market mean of intergenerational risk sharing. The lifetime utility of the household h is a function of instantaneous consumption: c_t^h in working period and d_{t+1}^h in retirement period. If we denote the time preferences by ρ with $\rho \geq 0$ and $\beta = \frac{1}{1+\rho}$ is the subjective discount factor, we have the following discounted lifetime utility function:

$$\bar{U}^h = U(c_t^h) + \beta U(d_{t+1}^h) \tag{3}$$

where the instantaneous utility is supposed to take the following form: $U(c) = \ln c$. The first and second period budget constraints are:

$$\begin{aligned} c_t^h + s_t^h &= (1 - \theta_u - \theta_q) w_t \\ d_{t+1}^h &= q_{t+1}^h + R_{t+1} s_{t+1}^h \end{aligned} \tag{4}$$

where $R_{t+1} = (1 + r_{t+1})$, $c_t^h, d_{t+1}^h \geq 0$, $s_t^h \geq 0$ and $s_{t+1}^h = 0$. The restriction on $s_{t+1}^h = 0$ implies that there is no bequest motive at the end of the life and $s_t^h \geq 0$ means that agents are liquidity constrained. The income of retirees q_t^h is defined as follows:

$$q_t^h = \begin{cases} b_t^u & \text{for } h = 2 \\ b_t^q & \text{for } h = 1 \end{cases} \tag{5}$$

A registered worker and unregistered worker caught in a random inspection will have equal income b_t^u with probability $m_t = (1 - \lambda_t) + \lambda_t f(\lambda_t)$. For the second case ($h = 1$), the probability is then $1 - m_t$. Agents solve an intertemporal maximisation problem subject to (4):

$$\underset{c_t^h, d_{t+1}^h}{Max} \left\{ U(c_t^h) + \beta U(d_{t+1}^h) \right\} \tag{6}$$

2.3 Social security

We consider a defined contribution unfunded (PAYG) scheme together with the informal support system defined by private transfers. We assume that the population growth is zero. Thus the size of consecutive generations is equal.

2.3.1 Formal social insurance - PAYG

PAYG scheme is self-financing where the contributions of registered workers pay the benefits of registered retirees. The budget constraint is then:

$$(\theta_u + \tau_w) m_t l_t^d w_t = m_{t-1} l_{t-1}^d b_t^u \tag{7}$$

where $m_t l_t^d$ is the share of registered workers contributing to pension scheme. The pension benefit is defined by $b_t^u = \varphi_t w_t$ thus the accrual rate φ_t is equal to $\frac{(\theta_u+\tau_w) m_t l_t^d + b_t^a}{m_{t-1} l_{t-1}^d}$. Note that, as such, public pension system includes both contributory and redistributive elements.

2.3.2 Informal social insurance - Social norm of elderly support

We suppose that working generations share their income at rate θ_q with those having no pension income. θ_q is determined by social consensus. The private transfer received by unregistered retirees b_t^q is given by the following constraint:

$$\theta_q l_t^d w_t = (1 - m_{t-1}) l_{t-1}^d b_t^q. \tag{8}$$

We see that this social consensus operates the same way as taxation for registered workers.

Now, we can define $Q_t^h = (1 - \theta_u - \theta_q) w_t + \frac{b_{t+1}}{R_{t+1}}$ as lifetime income of the household h. The lifetime income of registered workers is

$$Q_t^2 = (1 - \theta_u - \theta_q) w_t + \frac{b_{t+1}^u}{R_{t+1}} \tag{9}$$

where $b_{t+1}^u = \frac{(\theta_u+\tau_w) m_{t+1} w_{t+1}}{m_t}$ from pension scheme rule. The lifetime income of unregistered workers is

$$Q_t^1 = (1 - \theta_u - \theta_q) w_t + \frac{b_{t+1}^q}{R_{t+1}} \tag{10}$$

where $b_{t+1}^q = \frac{\theta_q w_{t+1}}{(1-m_t)}$ from private transfer rule.

2.3.3 Auditing

Auditing is a costly activity with constant returns to scale and the total cost $e(.)$ depends on the product of auditing frequency with total labour

demand reflecting the volume of auditing activity. The unit cost of auditing is χw_t, the total cost will be expressed as $e(z) = \chi w_t z l_t^d$. As such, the parameter χ expresses the efficiency of auditing. If χ decreases, auditing becomes less costly, thus more efficient.

We suppose that the cost of auditing activity is covered by collected penalties for employing unregistered workers and the auditing agency operates with a balanced budget:

$$f\lambda_t \phi w_t l_t^d - \chi z w_t l_t^d = 0 \tag{11}$$

which results the following relation: $\phi = \frac{\chi}{\lambda_t^2}$. In this context, the auditing agency can choose between two instruments: ***penalty levels*** or ***frequency of auditing*** to fight against unregistered employment. Note that changing the frequency of auditing is easier than changing penalty rate since the determination of a penalty rate requires a legal step however changing the frequency of auditing is an administrative measure. While we analyse the impact of both instruments, in fact governments mostly choose to tune with the frequency of auditing.

2.4 Markets

Each period, households' total savings is invested and determines next period capital stock. With full capital depreciation, we can express the capital market equilibrium as follows:

$$l_t^d(m_t s_t^2 + (1 - m_t)s_t^1) = k_{t+1}. \tag{12}$$

The labour market clearing condition implies that the demand for labour by the representative firm l_t^d equals the supply of labour by the representative household l_t^s. Goods market equilibrium can be written as follows:

$$y_t = l_t^d(m_t c_t^2 + (1 - m_t)c_t^1) + l_{t-1}^d(m_{t-1} d_t^2 + (1 - m_{t-1})d_t^1) + k_{t+1} \tag{13}$$

3 Equilibrium and steady state

Given a set of policy rules $\{\tau_w, \phi, z\}$, contribution rate to PAYG pension scheme and private transfer rate $\{\theta_u, \theta_q\}$ and an initial capital level $\{k_0\}$, probability of detecting informal employment $f(\lambda_t)$, an equilibrium for this

economy satisfies the following: the sequence of decision rules $\{c_t^{h,e},\ d_{t+1}^{h,e}\}$ solves consumer optimisation problem, the allocation rule $\{w_t^e,\ r_t^e,\ \lambda_t^e\}$ solves firms' maximisation problem and goods, capital and labour markets clear. Appendix A provides with the computation of equilibrium wage and capital path.

Next, we will analyse the impact of social norm of support, auditing policy and informal employment on welfare and growth at the steady state.

3.1 Impact of social norm

We consider a utilitarian social welfare function at steady state. The lifetime discounted utility of household h is denoted by $\bar{U}^h = U_i^h + \beta U_j^h$ where i and j refer to working and retirement period respectively. Social welfare is given as follows:

$$\begin{aligned} SW^* &= \bar{U}_j + \sum_{t=0}^{\infty} \beta^t \bar{U} \\ &= \bar{U}_j + \frac{1}{1-\beta}\bar{U} \end{aligned}$$

where $\bar{U} = \frac{m}{2}\bar{U}^2 + \frac{1-m}{2}\bar{U}^1$ is weighted generational lifetime discounted utility and $\bar{U}_j = \frac{m}{2}\bar{U}_j^2 + \frac{1-m}{2}\bar{U}_j^1$ is weighted retirement welfare[4].

Proposition 1 *1. Private transfers affect negatively steady state wage and capital:* $\frac{\partial w^*}{\partial \theta_q} < 0$ *and* $\frac{\partial k^*}{\partial \theta_q} < 0$.

2. There exists at least one private transfer rate maximising social welfare $\theta_q^s < 1 - \theta_u$ *if* $\left.\frac{\partial SW^*}{\partial \theta_q}\right|_{\theta_q=0} > 0$.

These results suggests that, in our economy, while there will be social welfare gains from mutual support, the wage level will be negatively affected. Given the norm for mutual support, lower levels of savings will lead to lower levels of capital, which in turn means lower growth. The private transfer rate has definitely a negative impact on growth but we should consider social welfare before making a policy statement on the existence of informal social

[4]Note that $\beta < 1$.

insurance mechanism. The impact of any change in private transfer rate on the steady state level of social welfare will be:

$$\frac{\partial SW^*}{\partial \theta_q} = \frac{1}{(1-\beta)\theta_q}\left(m\xi_{\theta_q}^{Q^2} + (1-m)\xi_{\theta_q}^{Q^1} + \frac{1}{2}\xi_{\theta_q}^{R}\right) \tag{14}$$

where ξ is the elasticity of the variable at the superscript with respect to private transfer rate θ_q and Q^h is steady state lifetime income. The second part of proposition (1) relates to the impact of private transfers on social welfare. Even though growth necessitates a decrease in private transfers, we see that a positive level of private transfer may be necessary for the optimisation of social welfare. The proof for the existence is provided in Appendix B.

We see that social cohesion decreases wage levels in the economy. This is a very familiar picture for underdeveloped and developing countries where public insurance system is coupled with informal insurance networks, generally based on familial bounds. In fact, such social support networks creates room for lower wage levels by reducing the stress of workers to insure several risks solely with their own income. Another impact of this social cohesion can be the difficulty to catch-up with developed countries as savings are done through private networks.

3.2 Impact of auditing policy

As auditing agency operates with balanced budget, we will obtain a level of auditing probability given penalty rate (given ϕ, $z^e = \frac{\tau_w+\theta_u}{2(\tau_w+\theta_u+\phi)}\sqrt{\frac{\phi}{\chi}}$) or a penalty rate given auditing frequency (given z, $\phi^e = (\tau_w+\theta_u)(-1+\frac{\tau_w+\theta_u}{8z^2\chi}(1-\sqrt{1-\frac{16z^2\chi}{\tau_w+\theta_u}}))$). Obviously, these instruments have a negative impact on wage and capital as they increase labour cost.

Auditing agency could determine ϕ to maximise steady state social welfare. The impact of a change in penalty rate on the steady state level of social welfare will be:

$$\frac{\partial SW^*}{\partial \phi} = \frac{1}{(1-\beta)\phi}\left(m\xi_{\phi}^{Q^2} + (1-m)\xi_{\phi}^{Q^1} + m\xi_{\phi}^{m}\ln\frac{Q^2}{Q^1} + \frac{1}{2}\xi_{\phi}^{R}\right) \tag{15}$$

where ξ is the elasticity of the variable at the superscript with respect to penalty rate ϕ. We see that the first part of this derivative $(m\xi_{\phi}^{Q^2} + (1-m)\xi_{\phi}^{Q^1})$

is negative as penalties have a negative impact on wage, but the second part is positive ($m\xi_\phi^m \ln \frac{Q^2}{Q^1} + \frac{1}{2}\xi_\phi^R$) as there will be a positive income effect in retirement thanks to decrease in informalisation through penalties and increase in interest rate. The calculation of this derivative does not give an explicit result as to the existence of an optimal level of detection penalty. We can simply state that in this economy, if the impact of increase in income outweighs the impact of penalties on wage, auditing agency can choose a positive level of penalty to control informal labour demand.

Another intuition regarding an optimal choice of penalty stems from short term analysis of welfare. It is obvious that an increase in penalty would immediately reduce working generation income and retired income through determination of elderly income by intergenerational distribution. There will be an increase in next retired generation income through formalisation but this will be outweighed by the decrease in wage. If auditing agency worked for a government which in turn had election concerns, this short term analysis suggests that there would be no penalties or very low level of penalties and government would follow a laissez-faire policy.

Remark 2 *If an economy is structured given a certain level of initial informalisation, (in our model generated by demand side cost reducing purposes), the economy might be trapped in persistent informality. In addition, fights against informality would have asymmetric consequences for different parts of the population i.e. formal agents would suffer from a decrease in wage levels but some informal would be better off after being caught and becoming formal. The structural problem can be remedied, in our opinion, by multi-layered and coordinated policies i.e. higher syndicate activities, effective and efficient enforcement, active labour market policies etc. as Bau (2021) demonstrates with an experimental study. In that case, private transfers serving for intergenerational distribution of wealth to insure income risk of informal retirees may be channelled to capital accumulation.*

3.3 Comparison with benchmark economies

We now consider two benchmark cases for this economy: the first (Economy 1) is an economy where all workers are registered ($\lambda = 0$) and there is no auditing agency and social support network ($\theta_q = 0$) and the second (Economy 2) is characterised by a non-negative share of unregistered employment ($\lambda > 0$) with auditing and no informal social insurance network ($\theta_q = 0$). We will

compare Economy 1 and 2 with the model economy to provide insights related to the effects of informality, private transfers and penalties.

If we note the steady state capital and wage levels in each economy by (k_1^*, w_1^*) and (k_2^*, w_2^*) then the following proposition will hold.

Proposition 3 *1.* $k_2^* > k^*$ *and* $w_2^* > w^*$
2. $k_2^* > k_1^*$ *and* $w_2^* > w_1^*$
3. $w_1^* < w^*$ *and* $k_1^* < k^*$ *if* $\theta_q < \bar{\theta}_q$
where $\bar{\theta}_q = \frac{(1-\theta_u)(1-m)(\theta_u+\tau_w)}{1+\tau_w}$.

Compared to golden rule of capital accumulation, all economies are dynamically inefficient due to the presence of social security institution. Still, we see that the steady state level of capital in Economy 2 is above the steady state capital in the model economy and Economy 1 as informal employment allows for cost reduction and there are incentives on the demand side to employ workers without registering. We see that the level of private transfers is crucial as private transfers should be below a certain limit to obtain a higher steady state capital. If there is a social norm requiring greater private transfers, the steady state capital level will be lower than the steady state with no informality. The social norm is so exigent that there is a pressure on capital accumulation.

The picture is not very clear when we look from welfare perspective. We have seen that a certain level of private transfers could be socially optimal. **The low frequency of auditing activities and low penalties in detection in underdeveloped and developing countries** could very well suggest that political authorities have particularly chosen this attitude given a strong social norm protecting retired generations.

4 Determination of the social norm

In this economy, only unregistered workers face the risk of having no pension benefit when they are retired and these workers will have an incentive to determine $\theta_q^{1,*} > 0$. We suppose that the individually optimal rate of private transfer is determined given the steady state of the economy. When a worker accepts to work informally, his expected retired utility accounting for the risk of being without social insurance when old is as follows: $u(\theta_q^1) = (1-f)\ln(\theta_q^1 \frac{w}{(1-m)R}) + f\ln((\theta_u+\tau_w+\frac{b^a}{m})\frac{w}{R})$. The maximisation of utility implies:

$\frac{\partial u}{\partial \theta_q} = (1-f)\frac{1}{\theta_q \frac{w}{(1-m)R}}\frac{\partial(\theta_q \frac{w}{(1-m)R})}{\partial \theta_q} + f\frac{1}{(\theta_u+\tau_w)\frac{w}{R}}\frac{\partial((\theta_u+\tau_w)\frac{w}{R})}{\partial \theta_q} = 0$. If we note the elasticity of discounted wage rate to private transfer rate by $\xi_{\theta_q}^{\frac{w}{R}} = \frac{\partial(\frac{w}{R})}{\partial \theta_q}\frac{\theta_q}{\frac{w}{R}}$, informal employees choose their individually optimal private transfer rate $\theta_q^{1,*}$ at $\xi_{\theta_q}^{\frac{w}{R}} = -(1-f)$. Young employees working formally would have no intention to provide transfers if only their choice was concerned $\theta_q^{2,*} = 0$ as these transfers decrease only income when young and do not generate any effect on lifetime utility and income when retired. We could argue that an average private transfer rate $\theta_q^* = (1-m)\theta_q^{1,*} + m\theta_q^{2,*}$ would be imposed by a social norm to support unregistered retirees with no income.

Proposition 4 *There exist a non negative socially determined private transfer rate i.e.* $\theta_q^* > 0$.

The proof for the existence is provided in Appendix C. How does socially determined level of private transfers affect welfare and growth? The upper bound for private transfers defined in Proposition 3 ($\bar{\theta}_q$) and the socially optimal level of private transfer (θ_q^s) can provide an intuition in this framework.

4.1 Comparison of θ_q^* with $\bar{\theta}_q$

If private transfer rate is less than the upper bound then, even though the economy allows for a certain level of informalisation, growth and wage rates would not be worse than the case with no informality. The following proposition gives the condition under which model economy will perform better than Economy 1 where there is not any informal employment.

Proposition 5 *There exists a lower bound* $\bar{\chi}$ *such that for all* $\chi > \bar{\chi}$, $\theta_q^* < \bar{\theta}_q$.

For a certain lower bound for efficiency of auditing activity $\bar{\chi}$ above which social security institution conducts costly auditing activity, the model economy will perform better than an economy with no informal employment.

4.2 Comparison of θ_q^* with θ_q^s

Proposition 4 suggests that there exists a non-negative private transfer rate that can be chosen as a social norm initially by the working generation. However, the transfer rate to uncovered retired generation socially acceptable

might be different than the socially optimal transfer rate θ_q^s. As, socially optimal transfer rate is obtained by maximisation of the utility of all generations of registered and unregistered households and as the utility of all registered and unregistered workers and registered retirees decreases with the level of transfers, the intuition is that the maximisation of social welfare will require a transfer level lower than socially determined rate $\theta_q^* > \theta_q^s$.

4.3 An illustration: Turkish case

We have used Turkish data to provide an illustration of above results. Turkish labour market is characterised by significant informality. Brook and Whitehouse (2006) suggests that in Turkey, the pension system is one of the barriers for formality as the high social security contribution rates discourage firms from employing low-skilled workers formally. In the last two decades, informality rate has declined from 50.1% in 2004 to 33.4% in 2018 while it remained stable from 2014 on. Bağır et al. (2021) find that half of the decline can be explained by the change in the workforce composition and industries.

In Turkey, employees contribute 9% (θ_u) of their gross wage and employers contribution is 11% (τ_w). Sevinç et al. (2021) provides an estimate of 8.3% for the real interest rate, the implied value of discount factor β is then 0.92. As we have mentioned above there is 33.4% informality in 2018. In accordance with an informal employment level of 33.4%, the penalty rate $\phi = 0.9$ with efficiency of auditing $\chi = 0.1$.

- Following Hall and Jones (1999), we have first taken α as 0.33. The social norm is found as $\theta_q^* = 0.08$ with $\bar{\theta}_q = 0.05$. There is a corner solution for socially optimal level of private transfers i.e. $\theta_q^s = 0$. Note that although welfare optimisation requires not to make any transfer at the steady state, there will be a positive transfer rate dictated by social norms. As $\theta_q^* > \bar{\theta}_q$, we have seen in Proposition 3 that this level will provide with an economy with lower growth than an economy with no informality. An increase in α to 0.5 will only change one result: $\theta_q^* = 0.07$.

- Now suppose that we have the same environment but production is less capital intensive with $\alpha = 0.25$. The increase in α will not change $\bar{\theta}_q = 0.05$ but the social norm will dictate a higher rate of private transfer $\theta_q^* = 0.083$ and moreover it will be socially efficient to make transfers even if at a lower rate than the social norm i.e. $\theta_q^s = 0.039 < \theta_q^*$. Note

that for this economy, we have $\theta_q^* > \bar{\theta}_q$ as well, this economy performs worse than an economy with no informality in terms of growth. However, this time, social welfare optimization requires a positive rate of private transfer.

These results are in line with the findings of Cox et al. (2006) study on private transfers using data of a set of 11 diverse developing countries. They find also that while in some countries private transfers flow from younger to older, in others older people make transfers to younger people. The former is suggested to arise from a potential interplay between private transfers and public pensions as generous public pension systems decreases private old age support. For the Latin and Central American countries where public pension do not provide enough for the elderly, the private transfer ratio to the mean income is around 0.055-0.065. Cox et al. (2006) argue that similar behavioural motives may cause similarities in private transfer making. Here, we provide a theoretical model from a behavioural perspective and show that the social consensus on an informal insurance mechanism for protecting informal working family members determine private transfers direction and magnitude.

5 Conclusion

We presented an economy characterised by informal employment and a social norm of elderly support. The equilibrium of the model suggests a positive level of private transfers affecting negatively the wage level in the economy. This result coincides with the fact that in the presence of social support networks, lower wage levels can be sustained in the labour market. When we compare this economy with the one without informal employment, we see that with informal employment alternative, labour costs are reduced and equilibrium wage increases to contribute to capital accumulation. Note that this result is relevant as long as the level of support to retirees dictated by the social norm is lower than the public insurance contribution rate. The introduction of random auditing only aggravates the situation by increasing the expected cost of labour. When we compare model economy with versions without informality and social network, we see that the possibility to under-declare number of workers and evasion from responsibility of social insurance may have positive impact on the economy in the presence of social support networks.

The fact that most governments do not prioritise fight against informalisation supports the results of the model. While this result can be very counter-intuitive from a fiscal policy stance, it underlines possible interference factors when a reduction in informal employment is targeted. If there are coexisting insurance mechanisms emerging from social norms to accommodate risks of informal employment in one hand and employing unregistered labour is a strategy to reduce labour cost on the other hand, public social and fiscal policies should simultaneously address labour tax fraud and substitute efficiently the social norm of mutual support with increased coverage of public insurance. Such actions appear more difficult when the economy suffer from many other structural economic problems (such as low value added production, high public deficits, high poverty and unemployment, chronic inflation problem) that prevent the government to attack informality issues at first hand. Allowing for informal insurance networks to make up for the structural problems would be a short-sighted side stepping strategy. While social support networks can be a desired form a social cohesion, right based public insurance can contribute to social and individual welfare more directly and efficiently through wider risk coverage and higher and longer term benefit provision. The predisposition of social norm of support could then be channelled towards other issues of social and individual well-being where the contribution of social bonding and collective action would be much more valuable and indeed this can not be achieved solely with government institutions.

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A Equilibrium

We can write first order conditions for profit maximisation as follows:

$$r_t + 1 = \alpha \frac{y_t}{k_t} \tag{16a}$$

$$\tilde{w}_t = (1-\alpha)\frac{y_t}{l_t^d} \tag{16b}$$

$$\frac{df}{d\lambda}\lambda_t + f = \frac{\tau_w + \theta_u}{\tau_w + \theta_u + \phi} \tag{16c}$$

where $\tilde{w}_t = w_t(1 + \tau_w + \lambda_t(f\phi - (1-f)(\theta_u + \tau_w)))$. Denote by $g_t = 1 + \tau_w + \lambda_t(f\phi - (1-f)(\theta_u + \tau_w))$. When we solve for equation (16c), we get the optimal level of informal labour demand given by equation: $\lambda^e = \frac{\tau_w + \theta_u}{2z(\tau_w + \theta_u + \phi)}$

and $g^e = 1 + \tau_w - \frac{(\tau_w+\theta_u)^2}{4z(\tau_w+\theta_u+\phi)}$. By constant returns to scale, there is normal profit and therefore $(\frac{1+r_t^e}{\alpha})^\alpha(\frac{\tilde{w}_t^e}{1-\alpha})^{1-\alpha} = 1$.

We can write first order conditions for household intertemporal maximisation as follows:

$$\frac{(c_t^h)^{-1}}{\beta(d_{t+1}^h)^{-1}} = R_{t+1} \tag{17}$$

We have to use the intertemporal budget constraint $c_t^h + \frac{d_{t+1}^h}{R_{t+1}} = (1 - \theta_u - \theta_q)w_t + \frac{q_{t+1}^h}{R_{t+1}}$ to get the level of expenditures and consumption. The optimal consumption schedules are $c_t^{h,e} = \frac{Q_t^h}{1+\beta}$ and $d_{t+1}^{h,e} = \frac{\beta R_{t+1} Q_t^h}{1+\beta}$ for working and retirement periods respectively.

The goods market equilibrium is then $m_t c_t^{2,e} + (1 - m_t)c_t^{1,e} + m_{t-1}d_t^{2,e} + (1 - m_{t-1})d_t^{1,e} + k_{t+1}^e = y_t^e$ where total private consumption demand is $\frac{m_t Q_{2,t}+(1-m_t)Q_{1,t}}{1+\beta} + \frac{\beta R_t(m_{t-1}Q_{2,t-1}+(1-m_{t-1})Q_{1,t-1})}{1+\beta}$, demand for capital good is $k_{t+1}^e = m_t s_t^{2,e} + (1 - m_t)s_t^{1,e} = (1 - \theta_u - \theta_q)w_t^e - \frac{m_t Q_t^2}{1+\beta} - \frac{(1-m_t)Q_t^1}{1+\beta}$. Equilibrium in capital markets implies:

$$(1 - \theta_u - \theta_q)w_{t-1}^e - \frac{m_{t-1}Q_{t-1}^2}{1+\beta} - \frac{(1-m_{t-1})Q_{t-1}^1}{1+\beta} = \frac{\alpha\tilde{w}_t^e}{(r_t^e+1)(1-\alpha)}.$$

Equilibrium in goods markets implies:

$$(1 - \theta_u - \theta_q)w_t^e + \frac{\beta R_t(m_{t-1}Q_{t-1}^2+(1-m_{t-1})Q_{t-1}^1)}{1+\beta} = \frac{\tilde{w}_t^e}{1-\alpha}.$$

We obtain the following equilibrium wage and capital dynamics using above equilibrium conditions:

$$w_t = (\frac{\alpha\beta(1 - \theta_u - \theta_q)g^{\frac{\alpha-1}{\alpha}}(1-\alpha)^{\frac{1}{\alpha}}}{(1+\alpha\beta)((\theta_u+\tau_w)m+\theta_q)+(1+\beta)\alpha(1-\theta_u-\theta_q)})^\alpha w_{t-1}^\alpha$$

$$k_t = \frac{\alpha(1-\alpha)\beta(1 - \theta_u - \theta_q)}{(1+\alpha\beta)((\theta_u+\tau_w)m+\theta_q)+(1+\beta)\alpha(1-\theta_u-\theta_q)}(k_{t-1})^\alpha$$

B Social welfare at steady state

At steady state, social welfare function becomes an infinite sum of constant lifetime welfare of all young generations plus welfare of the old generation alive at any period t:

$$SW^* = \frac{m}{2}\ln\frac{\beta RQ_2}{1+\beta} + \frac{1-m}{2}\ln\frac{\beta RQ_1}{1+\beta} + \frac{1}{1-\beta}\left(\frac{m}{2}\left(\ln\frac{Q_2}{1+\beta} + \beta\ln\frac{\beta RQ_2}{1+\beta}\right) + \frac{1-m}{2}\left(\ln\frac{Q_1}{1+\beta} + \beta\ln\frac{\beta RQ_1}{1+\beta}\right)\right)$$

$$SW^* = \frac{1}{1-\beta}\left(m\ln Q_2 + (1-m)\ln Q_1 + \frac{1}{2}\ln R + \frac{1}{2}\ln\beta + \ln\frac{1}{1+\beta}\right)$$

$$\frac{\partial SW^*}{\partial\theta_q} = \frac{1}{1-\beta}\left(m\frac{-w + (1-\theta_u-\theta_q)\frac{\partial w}{\partial\theta_q} + (\theta_u+\tau_w)\frac{\partial w/R}{\partial\theta_q}}{(1-\theta_u-\theta_q)w + \frac{(\theta_u+\tau_w)w}{R}}\right.$$
$$\left. + (1-m)\frac{-w + (1-\theta_u-\theta_q)\frac{\partial w}{\partial\theta_q} + \frac{w}{(1-m)R} + \frac{\theta_q}{1-m}\frac{\partial w/R}{\partial\theta_q}}{(1-\theta_u-\theta_q)w + \frac{\theta_q w}{(1-m)R}} + \frac{1}{2R}\frac{\partial R}{\partial\theta_q}\right)$$

We can analyse the sign of the derivative of social welfare with respect to private transfer rate to have an insight on the graph of social welfare. We refer to the value of R and w with respect to private transfer rate at $\theta_q = 1-\theta_u$ as R_1 and w_1.

$$\left.\frac{\partial SW^*}{\partial\theta_q}\right|_{\theta_q=1-\theta_u} = \frac{1}{1-\beta}\left(\frac{m\left(-w_1 + (\theta_u+\tau_w)\left.\frac{\partial w/R}{\partial\theta_q}\right|_{\theta_q=1-\theta_u}\right)}{\frac{(\theta_u+\tau_w)w_1}{R_1}}\right.$$
$$\left. + \frac{(1-m)\left(-w_1 + \frac{w_1}{(1-m)R_1} + \frac{1-\theta_u}{1-m}\left.\frac{\partial w/R}{\partial\theta_q}\right|_{\theta_q=1-\theta_u}\right)}{\frac{(1-\theta_u)w_1}{(1-m)R_1}} + \frac{1}{2R_1}\left.\frac{\partial R}{\partial\theta_q}\right|_{\theta_q=1-\theta_u}\right)$$

$$= \frac{R_1}{1-\beta} - \left(\frac{-m}{\theta_u+\tau_w} + \frac{(1-m)(1-R_1+mR_1)}{(1-\theta_u)R_1}\right.$$
$$\left. + \left.\left(\frac{R_1\partial w}{w_1\partial\theta_q} - \frac{\partial R}{R_1^2\partial\theta_q}\right)\right|_{\theta_q=1-\theta_u}\right) < 0$$

Since the derivative is negative at the upper limit of private transfers, the sufficient condition for the existence of a maximum in the interval $[0, 1-\theta_u)$ is $\left.\frac{\partial SW^*}{\partial\theta_q}\right|_{\theta_q=0} > 0$.

C Determination of private transfers

C.1 Existence

Agents working informally will determine $\theta_q^{1,*}$ where $\xi_{\theta_q}^{\frac{w}{R}} = -(1-f)$. Agents working formally choose $\theta_q^{2,*} = 0$ as these transfers decrease only income when young and do not generate any effect on lifetime utility and income when retired. As $\xi_{\theta_q}^{\frac{w}{R}} = \frac{\partial(\frac{w}{R})}{\partial\theta_q}\frac{\theta_q}{\frac{w}{R}}$, we have $\xi_{\theta_q}^{\frac{w}{R}} = -\frac{1}{1-\alpha}\frac{\theta_q}{R}\frac{\partial R}{\partial\theta_q}$. $\theta_q^{1,*}$ will be the solution of the following equation:

$$-\frac{1}{1-\alpha}\left.\frac{\theta_q}{R}\frac{\partial R}{\partial\theta_q}\right|_{\theta_q=\theta_q^{1,*}} = -(1-f) \implies \frac{1}{1-\alpha}\left.\frac{\theta_q}{R}\frac{\partial R}{\partial\theta_q}\right|_{\theta_q=\theta_q^{1,*}} - (1-f) = 0. \tag{18}$$

After arranging the terms of this equation, we obtain the following second degree polynomial: $A(\theta_q^{1,*})^2 + B\theta_q^{1,*} + C = 0$ where $A = (1-\alpha)^2(1-f)$, $B = -(2(1-f)(1-\alpha)^2(1-\theta_u) - (1+\alpha\beta)((\theta_u+\tau)m+1-\theta_u)((1-f)(1-\alpha)+1))$ and $C = -(1-f)(1-\alpha)(1-\theta_u)((1+\alpha\beta)((\theta_u+\tau)m+1-\theta_u) - (1-\alpha)(1-\theta_u))$ with $A > 0$ and $C < 0$. Remember that $\theta_q^* = (1-m)\theta_q^{1,*}$, so if $\theta_q^{1,*}$ is the root of the previous second order equation and $f(x) = Ax^2 + B(1-m)x + C(1-m)^2$ then θ_q^* will be the root of $f(x) = 0$. The product of roots of this equation $\frac{C(1-m)^2}{A}$ is negative so that there is one negative and one positive root and the value of $f(x)$ at $x = 1$ is positive, so there exists one positive root such that $\theta_q^* < 1$.

C.2 Comparison of θ_q^* with $\bar{\theta}_q$

We can use equation (18) and define $g(x) = \frac{1}{1-\alpha}\left.\frac{\theta_q}{R}\frac{\partial R}{\partial\theta_q}\right|_{\theta_q=x} - (1-f)$ to compare θ_q^* with $\bar{\theta}_q$. Remember that $\theta_q^{1,*}$ is the root of $g(x) = 0$.

Take $\theta_q^* = \frac{(1-\theta_u)(1-m)(\theta_u+\tau_w)}{1+\tau_w}$ then $\theta_q^{1,*} = \frac{(1-\theta_u)(\theta_u+\tau_w)}{1+\tau_w}$. If we evaluate $g(x)$ at $x = \frac{(1-\theta_u)(\theta_u+\tau_w)}{1+\tau_w}$. The function $g(x)$ is increasing with x such that if $g(\frac{\bar{\theta}_q}{1-m}) > 0$ we will have $\theta_q^{1,*} < \frac{\bar{\theta}_q}{1-m} \Rightarrow \theta_q^* < \bar{\theta}_q$.

We can introduce a new variable γ to define penalties as a function of contribution rates such that $\phi = (\tau_w + \theta_u)\gamma$[5]. This will simplify the analysis as well. $g(\frac{\bar{\theta}_q}{1-m})$ can be expressed as a function of γ where $g(\frac{\bar{\theta}_q}{1-m}) =$

[5] $z = \frac{1}{2(1+\gamma)}\sqrt{\frac{(\tau_w+\theta_u)\gamma}{\chi}}$, $f = \frac{1}{2(1+\gamma)}$ and $m = 1 - \frac{(1+2\gamma)}{2(1+\gamma)}\sqrt{\frac{\chi}{(\tau_w+\theta_u)\gamma}}$.

$\frac{1}{1-\alpha}\frac{1+\frac{\theta_u+\tau_w}{1-\theta_u}(1-\frac{(1+2\gamma)}{2(1+\gamma)}\sqrt{\frac{\chi}{(\tau_w+\theta_u)\gamma}})}{1-\frac{(1+2\gamma)}{2(1+\gamma)}\sqrt{\frac{\chi}{(\tau_w+\theta_u)\gamma}}+\frac{(1-\theta_u)(-(1-\alpha)(1-\theta_u)+(1+\alpha\beta)(1+\tau_w))}{(\theta_u+\tau_w)(1+\beta\alpha)(1+\tau_w)}} - \frac{(1+2\gamma)}{2(1+\gamma)}$. This condition is met if $\chi > \bar{\chi}$ where $\bar{\chi} = \frac{(1-\alpha)\frac{(1+2\gamma)}{2(1+\gamma)}(1+\frac{(1-\theta_u)(-(1-\alpha)(1-\theta_u)+(1+\alpha\beta)(1+\tau_w))}{(\theta_u+\tau_w)(1+\beta\alpha)(1+\tau_w)})-1-\frac{\theta_u+\tau_w}{1-\theta_u}}{(1-\alpha)\frac{(1+2\gamma)}{2(1+\gamma)}-\frac{\theta_u+\tau_w}{1-\theta_u}}$. This means that there is a lower bound for efficiency of auditing activity.