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THE IMPACT OF SANCTIONS AND PRE-RELEASE POLICIES ON RECIDIVISM

MASTER'S THESIS

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SEPTEMBER 2022

Preface

This thesis is inspired by a simple question, what is the cause of amnesties? Is our justice just not enough? So that it requires some corrections on the way? Or is it an economical, therefore a political decision? Most of the amnesties we encountered in history were based on this economical approach where the convicted criminal population passed beyond the institutions' capacity. Or worse it was just granted because of a political promise. But what was the cost of this action to the rest of the population, where we encounter all sorts of crime daily.

Criminal economical theory has a long history with finding optimal solutions to crime and punishment, and explaining reasons of crime, starting with Becker's first research in 1976. But in this research our aim is understanding the population. In this direction, we used evolutionary game theory, and based our model on researches of other scientists such as John Maynard Smith, or Jorgen W. Weibull. We also used computer simulations written in Python to simulate every agent's action individually in a population.

I would like to thank my counselor Doç. Dr. Bilge Öztürk Göktuna for her contributions all over the thesis (especially in Chapter 3). I also thank my parents and my wife Alara for their endless support.

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Abstract

The maxim first stated by roman philosopher Cicero on his work "*De Legibus*" (106 BC), "Noxiae poena par esto" *aka* "Let the punishment fit the crime" quickly became one of the most important principle of criminal law. Said principle attract attention from various disciplines such as laws and economics. Though it is not always possible to hold that principle for all circumstances due to human error, there are some other mechanisms in criminal law used to approximate this standard like pardons, amnesties and paroles. But how does granting such early releasing policies affects general population? Does it change criminal behavior? If so, to which direction? Does non-criminal population also influenced by it? Does it alter recidivism rates?

Earliest studies on criminal economic theory revolved around this principle, Becker (1968), considered as one of the first works, inquired this principle from an economical side, and tried to find an optimal solutions to both crime and expenditures to prevent and punishing crimes. Later works (Ehrlich, 1973; Heineke and Block, 1975) mostly focused on same static problem, Block and Lind (1975), Schmidt and Witte (1984) and Witte (1980) leaned on understanding deterrence effect on crime, Levitt (1996) and Drato et al. (2009) analyzed and included incarceration effect to literature. On the dynamic side of the problem Leung (1995) formulated a dynamic deterrence model, Later on, Burdett et al. (2003) used search theory make a dynamic model and Mocan et al. (2005) proposed a dynamic model of criminal activity by introducing legal and criminal human capital. In this study, it is aimed to approach the same problem using game theory, evolutionary game theory in particular to not only understanding crime and suggesting optimal solutions to the problem also conceptualizing early release policies (amnesties, pardons and paroles) to apprehend its effect on population. Like Leund (1995), this study interested in recidivism to measure success of these policies, the main reason this study uses evolutionary game theory as a tool coincides with this statement, to measure recidivism it's required both more than one period and population dynamics so that agents can change their strategies.

To the requirements of having more than one period and population dynamics, the model is established on classic Hawk-Dove game, first introduced to literature by John Maynard-Smith and George Price (1973), merged with imitation dynamics (Weibull, 1995; Björnerstedt and Weibull, 1996; Hofbauer and Sigmund, 2003). Model considers a society of Hawks and Doves which encouraged be-

ing co-operative by declaring Hawks as criminals. Similar to original Hawk and Dove game; in a Hawk and Hawk encounter both agents share the resource (v), and wound each other (c), which makes the both agent's payoff $\frac{v}{2} - c$, if a Hawk and Dove encounters each other, Hawk gets all resource v, while Dove gets 0 and in the last case where Dove meets Dove in an encounter Doves share the resource and since there is no aggression between them they both gain $\frac{v}{2}$ as payoff. We suppose that the social norm is to act as a Dove; consequently acting as a Hawk is a crime. The criminals acting as a Hawk can be caught with a certain probability, and punished with imprisonment, which prevents the convicted agent from participating further games until the agent is released from conviction if there is a chance of amnesty. In addition, agents following the norms of the society must pay a fee from their payoffs like an income tax which is used for keeping the juridical system and incarceration institutes alive and protecting the innocents, and directly affects detection rates. We also assume there are decreasing returns to scale in the conviction process and this process works with efficiency rate ϵ.

When a Hawk player, criminal is detected and convicted, he is prevented from participating further games until his release if there is amnesty, for the period of imprisonment agent gets 0 as payoff. The convicted agent must wait for a pardon or amnesty to be released into the conventional population again. After every round, both the non-convicted population and newly-released agents can change their strategy by imitating other agents. The change depends on how good they performed in the round compared to a random agent in the population, which consists of both non-convicted and convicted agents. If an agent's latest payoff is greater than or equal to the randomly selected agent's, he does not change his strategy, otherwise he matches its strategy to the randomly selected agent's.

In this study, three different cases are analyzed to understand the effect of amnesties on criminal activity over population. First case is an iterated hawk and dove model without amnesty and with a conviction mechanism. Second case is where amnesty is introduced to the model with an exogenous probability. And in the last case, a simulation is used to visualize and explain models the results such as population distribution of behaviours, population's conviction status mix, probability of detection and recidivism rate. Follow up to the last case, there is a small subsection about interaction between specific variables.

In the first case without amnesty and second case with amnesty, the aim is finding steady states of the imitation dynamics. There are two types of steady states: a pure steady state where all the population adopt the same behaviour and a mixed state where different behaviours can be seen. The steady state is achieved when the population distribution of behaviours stops changing. In both cases, to find mixed steady states, three different configurations are used. While in all configurations we have $\phi = 0.02$ and the cost of confrontation compared with the value of the resource change ($v = c$, $v = 2c$, and $v = 3c$).

In this case criminality or playing against the norms of the population can be stable if the value of the resource is high enough ($v > 2c$). But there is a chance the case where the population as a whole act according to the social norm to be stable too, if the value of ϵ is sufficiently high, or the detection process is efficient enough. Although, there are some stable mixed states in the case where resource does not cover the cost of confrontation.

With the introduction of amnesties to the model, expected utility of criminal agents changes the most. The stability of the state with all playing Hawk or against norm of the society, stays the same. Any increase in the probability of an amnesty α, also increases the share of Hawks in the mixed states. Like previous cases, any scenario where $v < 2c$, also lowers the share of criminals in the mixed state.

In the last section, an agent-based simulation is created and used to visualize and calculate all the cases above. Which further proves and strengthens the propositions. The simulation used in this section consists of phases which represent the mechanics introduced in the model. The simulation iterates over some specific phases to imitate periods of time. In the first part simulation retrace the steps of previous cases and calculates respective recidivism rates. Therefore, it can be detected how any change in variables affects recidivism rates. The pairwise matching of agents in game events is randomized. This ensures that all games in the same period are unique and all agents gain one payoff at the end of the round. In case of an odd numbered non-convicted population, since convicted agents can not participate in the games, the last agent always gets the Dove versus Dove result, which is $\frac{v}{2}$. In all simulations, the initial state of variables is the same, except for the last subsection where the effects of different values of exogenous variables are checked on recidivism rate and final population distribution. These initial states include; 100 agents in total population, 20% hawk rate in initial population, 100 periods, $v = 10$, $\phi = 0.02$, amnesty rate $\alpha = 0.01$, and the efficiency of detection process $\epsilon = 0.01$.

In the simulation section, results are consistent with our previous sections.

Here, it can be deduced, that even a smaller chance of amnesty, increases both criminal activity and recidivism rate over population, but both can be stabilized, or reduced by an efficient detection policy. It is also noted, that further amnesty rates over 10% with given conviction probability does not directly increases crime rates but instead it spreads criminality over the population where every agent becomes a criminal at some period of time and gets convicted. The better results might be acquired with adding segmentation or network mechanisms to the model.

Two main outcomes can be extracted from this study. The first outcome is that any kind of deterrence effect created by incarceration policies protects the norm compliance population, even further, with efficient detection and conviction policies, it makes following the norm thrive over criminality. The second outcome is, that the slightest occurrence of amnesty can disturb and revert the deterrence effect created by incarceration policies.

In Turkey's recent history, over 60 pardons, of which 9 of them can be qualified as amnesties, were issued by the General Assembly of Turkish Republic. All of them have been issued due to some political reasons, these pre-release laws might have corrected some unjustified convictions, but their cost to the populations is higher than meets the eye. If the goal is correcting previous laws errors, an amnesty or a pardon might not be the correct policy. Efficient and better detection and conviction policies would prevent such unjustified convictions and protect the innocent population.

Résumé

La maxime énoncée pour la première fois par le philosophe romain Cicéron dans son ouvrage "*De Legibus*" (106 av. J.C.), "Noxiae poena par esto" alias "Que la peine corresponde au crime" est rapidement devenue l'un des principes les plus importants du droit pénal. Ce principe attire l'attention de diverses disciplines telles que le droit et l'économie. Bien qu'il ne soit pas toujours possible de maintenir ce principe pour toutes les circonstances dues à une erreur humaine, il existe d'autres mécanismes en droit pénal utilisés pour se rapprocher de cette norme, comme les pardons, les amnisties et les libérations conditionnelles. Mais comment l'octroi de telles politiques de libération précoce affecte-t-il la population générale ? Change-t-il le comportement criminel ? Si oui, vers quelle direction ? La population non criminelle en est-elle également influencée ? Cela modifie-t-il les taux de récidive ?

Les premières études sur la théorie économique criminelle tournaient autour de ce principe, l'étude de Becker (1968), considérée comme l'un des premiers travaux, a enquêté sur ce principe d'un point de vue économique et a tenté de trouver une solution optimale à la fois au crime et aux dépenses pour prévenir et punir les crimes. Les travaux ultérieurs (Ehrlich, 1973 ; Heineke et Block, 1975) se sont principalement concentrés sur le même problème statique, Block et Lind (1975), Schmidt et Witte (1984) et Witte (1980) se sont appuyés sur la compréhension de l'effet dissuasif sur le crime, Levitt (1996) et Drato et al. (2009) ont analysé et inclus l'effet de l'incarcération dans la littérature. Du côté dynamique du problème, Leung (1995) a formulé un modèle de dissuasion dynamique. Plus tard, Burdett et al. (2003) ont utilisé la théorie de la recherche pour créer un modèle dynamique et Mocan et al. (2005) propose un modèle dynamique de l'activité criminelle en introduisant le capital humain légal et criminel. Dans cette étude, l'objectif est d'aborder le même problème en utilisant la théorie des jeux, la théorie des jeux évolutionnaires, en particulier pour non seulement comprendre le crime et suggérer des solutions optimales au problème, mais aussi conceptualiser les politiques de libération anticipée (amnisties, grâces et libérations conditionnelles) pour appréhender son effet sur la population. Comme Leund (1995), cette étude s'est intéressée à la récidive pour mesurer le succès de ces politiques, la principale raison pour laquelle cette étude utilise la théorie des jeux évolutionnaires comme outil coïncide avec cette affirmation, pour mesurer la récidive, il faut à la fois plus d'une période et la dynamique de la population pour que les agents puissent

changer leurs stratégies.

Aux exigences d'avoir plus d'une période et de la dynamique des populations, le modèle est établi sur le jeu classique de Faucon et Colombe, introduit pour la première fois dans la littérature par John Maynard-Smith et George Price (1973), fusionné avec la dynamique de l'imitation (Weibull, 1995 ; Björnerstedt et Weibull, 1996 ; Hofbauer et Sigmund, 2003). Le modèle considère une société de Faucon et Colombe qui partageant une ressource commune. Similaire au jeu original Faucon et Colombe ; dans une rencontre Faucon et Faucon, les deux agents partagent la ressource (v) et se blessent (c), ce qui rend le gain des deux agents $\frac{v}{2} - c$, si Faucon et Colombe se rencontrent, Faucon obtient toutes les ressources (v), tandis que Colombe obtient 0 et dans le dernier cas où Colombe rencontre Colombe lors d'une rencontre, les Colombes partagent la ressource et puisqu'il n'y a pas d'agression entre elles, elles gagnent toutes les deux $\frac{v}{2}$ comme récompense. La norme sociale est supposée d'agir comme une Colombe donc agir comme un Faucon est une crime. Ces criminels peuvent être attrapés avec une certaine probabilité et punis d'emprisonnement, ce qui leur empêche de participer à d'autres jeux jusqu'à ce qu'ils soient libérés de leur condamnation, ce qui arrive également par hasard. De plus, les agents qui suivent les normes de la société doivent payer une redevance sur leurs gains, comme un impôt sur le revenu. La somme de ces frais est utilisée pour maintenir en vie le système judiciaire et les instituts d'incarcération, et affecte directement les taux de condamnation. Nous supposons également qu'il y a des rendements d'échelle décroissants dans le processus de condamnation, ϵ est l'efficacité du processus de détection.

Lorsqu'un Faucon, criminel est détecté et condamné, il est empêché de participer à d'autres jeux jusqu'à sa libération, pour cette période de condamnation, l'agent obtient 0 comme gain. Il n'y a pas de peine de prison prédéterminée, donc un agent condamné doit attendre une grâce ou une amnistie pour être à nouveau libéré dans la population conventionnelle.

Après chaque tour, la population non condamnée et les agents nouvellement libérés peuvent changer leur stratégie en imitant d'autres agents. Le changement dépend de la qualité de leurs performances dans le tour par rapport à un agent aléatoire dans la population, qui se compose à la fois d'agents non condamnés et condamnés. Si le dernier gain d'un agent est supérieur ou égal à l'agent aléatoire qu'il choisit, il ne change pas sa stratégie, au contraire il fait correspondre sa stratégie à celle de l'agent aléatoire.

Dans cette étude, trois cas différents sont analysés pour comprendre l'effet des amnisties sur l'activité criminelle au sein de la population. Le premier cas est un modèle itératif du faucon et colombe, sans amnistie mais avec un mécanisme de condamnation. Le deuxième cas est celui où l'amnistie est introduite et dans le dernier cas, une simulation est utilisée pour visualiser et expliquer les résultats des modèles tels que le mélange de stratégies de la population, le mélange de statuts de condamnation de la population, la probabilité de détection et le taux de récidive. Suite au dernier cas, il y a une petite sous-section sur l'interaction entre des variables spécifiques.

Dans le premier cas et le deuxième cas, l'objectif est de trouver des états stables pures ou mixtes. L'état d'équilibre est atteint lorsque la population cesse de changer. Dans les deux cas, pour trouver des états stationnaires mixtes, trois configurations différentes sont utilisées. Alors que toutes les configurations ont un taux de frais de $\phi = 0.02$ le rapprt entre le coût de confrontation et la valeur de la ressource change ($v = c$, $v = 2c$ et $v = 3c$).

Dans ce cas, la criminalité ou le jeu contre les normes de la population peut être stable si la valeur de la ressource est suffisamment élevée ($v > 2c$). Mais il y a une chance pour que les Colombes envahissent la population, si la valeur de ϵ est suffisamment élevée, ou si le processus de détection est suffisamment efficace. Cependant, il existe des stratégies mixtes stables dans le cas où les ressources ne couvrent pas le coût de la confrontation.

Avec l'introduction des amnisties dans le modèle, l'utilité espérée des agents criminels change le plus. La stabilité pure est plus forte, dans le cas de l'état pure avec toute la population jouant contre la norme de la société. Toute augmentation de la probabilité d'une amnistie α, augmente également la part des Faucons dans les états mixtes. Comme les cas précédents, tout scénario où $v < 2c$ réduit également la proportion de criminels dans un état mixte.

Dans la dernière section, une simulation est utilisée pour visualiser tous ces cas ci-dessus. Ce qui prouve et renforce encore plus les propositions. La simulation utilisée dans cette section est constituée de phases qui représentent les mécanismes introduits dans le modèle. La simulation itère sur certaines phases spécifiques pour imiter des périodes de temps. Dans la première partie, la simulation retrace les étapes des cas précédents et calcule les taux de récidive respectifs. Par conséquent, il est possible de détecter l'incidence de tout changement de variable sur les taux de récidive. L'appariement par paires des agents dans les événements de jeu est randomisé. Cela garantit que tous les jeux de la même période sont uniques et que tous les agents gagnent seulement un gain à la fin du tour. Dans le cas d'une population impaire de non-condamnés, puisque les agents condamnés ne peuvent pas participer aux jeux, le dernier agent obtient toujours le résultat Colombe vs Colombe, qui est $\frac{v}{2}$. Dans toutes les simulations, l'état initial des variables est le même, sauf la dernière sous-section où l'effet des différentes valeurs des variables exogènes sur le taux de récidive et la distribution de la population finale est analysé en changeant ces valeurs. Ces états initiaux comprennent ; 100 agents dans la population totale, 20% taux de faucon dans la population initiale, 100 périodes, $v = 10$, $\phi = 0.02$, taux d'amnistie $\alpha = 0.01$ et et efficacité du processus de détection $\epsilon = 0.01$.

Dans la section de simulation, les résultats sont cohérents avec nos sections précédentes. Ici, on peut en déduire, que même une moindre chance d'amnistie, augmente à la fois l'activité criminelle et le taux de récidive dans la population, mais les deux peuvent être stabilisés ou réduits par une politique de détection efficace. Il est également à noter que des taux d'amnistie supérieurs à 10% avec une probabilité de condamnation donnée n'augmentent pas directement les taux de criminalité, mais propagent plutôt la criminalité dans la population où chaque agent devient un criminel à un certain moment et est condamné. Les meilleurs résultats pourraient être obtenus en ajoutant des mécanismes de segmentation ou de réseau au modèle.

Deux résultats principaux peuvent être obtenus avec cette étude. Le premier est que tout type d'effet de dissuasion créé par les politiques d'incarcération protège la population conforme aux normes, encore plus, avec des politiques de détection et de condamnation efficaces, il fait que le respect de la norme prospère sur la criminalité. Le deuxième est que la moindre occurrence d'amnistie peut perturber et inverser l'effet dissuasif créé par les politiques d'incarcération.

Dans l'histoire récente de la Turquie, plus de 60 grâces, dont 9 peuvent être qualifiées d'amnisties ont été délivrées par l'Assemblé Nationale de République Turque. Toutes émises pour des raisons politiques, ces lois de pré-libération ont peut-être corrigé certaines condamnations injustifiées, mais leur coût pour les populations est plus élevé qu'il n'y paraît. Si l'objectif est de corriger les erreurs des lois antérieures, une amnistie ou une grâce pourrait ne pas être la bonne politique. Des politiques de détection et de condamnation efficaces et améliorées empêcheraient de telles condamnations injustifiées et protégeraient la population innocente.

Özet

Romalı filozof Cicero'nun "*De Legibus*" (MÖ 106) adlı eserinde ilk kez dile getirdiği "Noxiae poena par esto" namı diğer "Cezanın suça uymasına izin ver" sözü, kısa sürede ceza hukukunun en önemli ilkelerinden biri haline geldi. Söz konusu ilke, hukuk ve ekonomi gibi çeşitli disiplinlerden ilgi görmektedir. Bu ilkeyi insan hatası nedeniyle her koşulda tutmak her zaman mümkün olmasa da, ceza hukukunda bu standarda yaklaşmak için kullanılan af ve şartlı tahliye gibi başka mekanizmalar da vardır. Fakat bu tür erken tahliye politikaları genel nüfusu nasıl etkiler? Suç davranışlarını değiştirir mi? Eğer öyleyse, hangi yönde bir değişim olur? Suçlu olmayan nüfüs da bu politikalardan etkilenir mi? Yeniden suç işleme oranlarını değiştirir mi?

Suç iktisat teorisine dair erken çalışmalar bu ilke üzerine olmuştur. İlk çalışmalardan biri olarak kabul edilen Becker (1986) çalışması, bu ilkeyi ekonomik yönden sorgulamış ve hem suça hem de suçları önlemeye ve cezalandırmaya yönelik harcamalara optimal çözümler bulmaya çalışmıştır. Daha sonraki çalışmalar (Ehrlich, 1973; Heineke ve Block, 1975) çoğunlukla aynı statik soruna odaklanmıştır, Block ve Lind (1975), Schmidt ve Witte (1984) ve Witte (1980) cezanın suç üzerindeki caydırıcılığın etkisini anlamaya yönelirken, Levitt (1996) ve Drato vd. (2009) hapsedilme etkisini incelemiş ve literatüre dahil etmiştir. Sorunun dinamik tarafında Leung (1995) dinamik bir caydırıcılık modeli formülüze etmiştir, Daha sonra, Burdett vd. (2003), arama teorisini kullanarak dinamik bir model oluşturmuş ve Mocan vd. (2005), yasal ve cezai insan sermayesini ortaya atarak dinamik bir suç faaliyeti modeli önermiştir. Bu çalışma aynı soruna oyun teorisi, özellikle evrimsel oyun teorisi kullanılarak yaklaşılması, sadece suçu anlamak ve soruna optimal çözümler önermekle kalmayıp, bunun nüfus üzerindeki etkisini anlamak için erken tahliye politikalarını (af, af ve şartlı tahliye) kavramsallaştırmayı amaçlamaktadır. Leund (1995) çalışmasında olduğu gibi bu çalışma da suç tekrarı oranları üzerine eğilmiştir. Bunun sebebi bahsi geçen politikaların başarısının bu oranla ölçülebilmesidir. Çalışmanın ana aracının evrimsel oyun teorisi olması da yine bu sebepledir. Çünkü suç tekrarı oranlarının hesaplanabilmesi için bir periyottan fazla bir süreye ve popülasyonun stratejilerini değiştirebilecekleri bir çevreye ihtiyaçları vardır.

Bu gereksinimlerin karşılanması için model, ilk olarak John Meynard-Smith ve George Price (1973) tarafından literatüre kazandırılan Hawk-Dove oyununu imitasyon mekanikleri (Weibull, 1995; Björnerstedt ve Weibull, 1996; Hofba-

uer ve Sigmund) ile birleştirmiştir. Model şahinler ve güvercinlerden oluşan bir toplum üzerine kurulmuştur. Modelde kooperasyon, şahinleri suçlu ilan ederek teşvik edilmektedir. Orjinal Şahin ve Güvercin oyununda olduğu gibi, Şahin ve Şahin karşılaşmalarında, iki oyuncu kaynağı v ortak şekilde paylaşır ve birbirlerini c kadar yaralar, bu durumda her iki oyuncu da $\frac{v}{2} - c$ kadar kazanç sağlamış olur. Şahin ve Güvercin karşılaşırsa, Şahin bütün kaynağı (v) alırken, GÜvercin bir kazanç sağlamaz. Son senaryo olan Güvercin ve Güvercin karşılaşmasında ise iki oyuncu da kazancın yarısını $\frac{v}{2}$ alır. Modelde baroşcıl Güvercin stratejisi sosyal norm olarak alınmış ve Şahin stratejisi suş teşkil etmektedir. Şahin stratejisini güden oyuncular, belirli bir olasılık ile yakalanabilir ve ceza olarak hapse atabilirler. Hapis süresince gelecek oyunlara katılamazlar ve bir önceki turda aldıkları kazanç sıfırlanır. Bu süreç boyunca her tur sonunda sıfır kazanmaya devam ederler. Hapis süresi, genel af ilan edilmediği sürece sınırsızdır. Şahin oynamayan, normları takip eden oyuncular ise kazançlarının bir kısmını, gelir vergisi gibi toplayarak bir havuz oluşturur. Yargı ve hapishane sisteminin devamlılığı bu havuzda toplanan bütçeye bağlıdır ve doğrudan yakalanma oranı üzerinde etkisi vardır. Modelde, ölçeğe göre azalan getiri ile işleyen yakalama süreci, (ϵ) etkinliği ile işler.

Her dönem sonunda, hem serbest oyuncular hem de serbest olmayan oyuncular davranış seçimlerini diğer oyuncuları taklit ederek değiştirebilirler. Bu değişim, geçen dönem aldıkları kazancın, toplulukdan rastgele seçilen başka bir oyuncunun kazancından ne kadar iyi olduğuna bağlıdır. Eğer kazançları, rastgele seçilen oyuncunun kazancından yüksekse ya da eşitse stratejilerini değiştirmezler, aksi durumda stratejilerini seçilen oyuncunun stratejisi ile aynı hale getirirler. Bu süreçte seçilen rastgele oyuncu hapishanedeki bir oyuncu olabilir.

Çalışmada üç ayrı durum incelenmiş ve genel afların toplumdaki suç etkinliği ve nihai topluluk dağılımı üzerindeki etkisi anlaşılmaya çalışılmıştır. İlk ekonomik modelde tekrar eden bir şahin ve güvercin oyunu, genel af olmadan fakat hapis mekanizması olan bir çerçevede incelenir. İkinci oyunda genel af mekanizması, belirli bir orana α bağlı olacak şekilde modele eklenir. Son aşamada ise bir simülasyon kullanılarak, toplumun davranış dağılımı, yasal durumu karması (suçlu veya suçsuz), yakalnama oranlarına bağlı olarak görselleştirilir ve tekrar suç işleme oranlarını incelenir. Son aşamaya ek olarak dışsal değişkenlerin farklı değerlerinin, içsel değişkenleri nasıl etkilediğine dair bir alt bölüm daha vardır.

İlk ve ikinci bölümün amacı var olan davranış seçimlerinin durağan durumlarını bulmaktır. Bu durağan duruma, toplumun taklit yolu ile değişimi durduğu noktada ulaşılır. Burada iki durağan türü bulunur: toplumdaki tüm bireylerin aynı davranışı seçtiği *saf durum* ve toplumda farklı davranışların görüldüğü *karma durum*. Her iki bölümde de, karma durumları örneklemek için üç ayrı konfigürasyon kullanılmıştır. Bunların her birinde, alınan vergi (ϕ) 0.02 olarak belirlenmiş, ve kaynağın suçun maliyetini karşılama durumuna göre üç farklı örnek ($v = c$, $v = 2c$ ve $v = 3c$) ele alınmıştır.

İlk bölümde suçlu olma ya da toplumun normlarına karşı oynama, eğer kaynak suç maliyetinden yeteri kadar yüksekse ($v > 2c$) saf bir denge olabilir. Toplumun normalara uygun yaşadığı bir saf durağan denge yeteri kadar yüksek ϵ değerinde, bir başka deyişle yakalanma sürecinin yeteri kadar efektif olduğu durumda ortaya çıkar. Buna karşın, kaynağın maliyeti karşılamaya yetmediği durumlarda çeşitli karma dengeler de mevcuttur, fakat bu durumların hepsi stabil değildir.

Genel affın modele eklenmesi, en büyük etkiyi toplumun normlarına karşı oynayan şahin oyuncularının beklenen kazançları üzerinde gösterir. Bu strateji, en güçlü saf strateji olmaya devam eder. Genel af olasılığının (α) sisteme eklenmesi ya da bu orandaki en ufak bir artış bile karma durumlarda suç oranını artırır.

Son bölümde, az önce üzerinden geçtiğimiz durumları hesaplamak ve görselleştirmek için oyuncu bazlı bir simülasyon kurulmuş ve kullanılmıştır. Simülasyon sonuçları, bölümlerde vermiş olduğumuz ifadeleri güçlendirir yöndedir. Kullanılan simülasyon fazlardan oluşup her faz modelde tanıttığımız bir mekaniğe denk gelir. Simülasyon bu fazların bazıları üzerinde iterasyon yaparak, dinamik bir şekilde çalışır.

İlk parçada, simülasyon geçmiş bölümlerde yaptığımız adımları takip ederek, tekrar suç işleme oranlarını hesaplar. Bu sayede diğer dışsal değişkenlerin, tekrar suç işleme oranları üzerindeki etkileri tespit edilebilir. Oyuncuların ikili eşleşmelerinde şöyle bir süreç kullanılır: öncelikle serbest olan oyunculardan bir sepet oluşturulur ve bu sepet karıştırılır; daha sonra bu sepet içinden ikili kombinasyonlar oluşturulur. Böylelikle bir dönem boyunca oynanan tüm oyunların benzersiz olması ve hiç bir oyuncunun dönem sonunda birden fazla kazanç almaması garantilenmiş olur. Oyuncu sayısının tek sayı olduğu durumlarda ise, sona kalan oyuncu Güvercin ve Güvercin karşılaşmasının sonucu olan $\frac{v}{2}$ kazancını alır. Kullanılan tüm simülasyonlarda, başlangıç durumları aynı kabul edilmiştir, sadece son parçada farklı dışsal değişkenlerin, tekrar suç işleme oranları ve nihai topluluğa etkilerinin ölçülmüş, bu nedenle seçilen dışsal değişkenlerin farklı değerleri üzerinde iterasyon yapılmıştır. Bahsi geçen ilk durum ise şu şekilde belirlenmiştir: oyuncuların %20'si Şahin oyuncusu olacak şekilde, 100 oyuncu, 100 dönem iterasyon süresi, $v = 10$, kesinti $\phi = 0.02$, genel affın çıkma olasılığı %10, ve tespit sürecinin etkinliğ $\epsilon = 0.01$.

Bu bölümdeki sonuçlar, geçtiğimiz bölümlerde bulunan sonuçlar ile uyumludur. İkinci bölümde yaptığımız tespite ek olarak, burada en ufak bir af ihtimalinin bile hem toplum içindeki suç aktivitesini hem de tekrar suç işleme oranlarını artırdığı görülebilir. Fakat bu artışlar, tespit etkinliği ve yargılama süreçleri ile dengelenebilir. Simülasyon sonuçlarından çıkan bir diğer sonuç ise, %10 üzerindeki genel af çıkma oranı doğrudan toplumdaki suç oranını değiştirmese de, suç işleme stratejisini toplum üzerinde yaygınlaştırır. Örneğin yapılan bir simülasyonda, toplumdaki oyuncuların 100 dönem üzerinde ortalama 33 kere toplum normlarına karşı davranış sergilediği ve yine 100 dönemde ortalama 3 kere bu durumun tespit edilip oyuncunun yakalandığı gözlemlenmiştir. Bu süreçte toplumdaki oyuncuların birbirlerine yakınlıklarının simetrik olmasının payı yüksektir. Gelecek araştırmalarda ağ teorisinin ya da nüfus üzerinde segmentasyon olan bir modelin kullanılması, bu fenomeni açıklamaya yardımcı olabilir.

Bu çalışmadan iki ana çıkarım yapmak mümkündür. İlk çıkarım, caydırıcılık etkisi taşıyan hapis politikaları toplum normlarını takip eden oyuncuları koruduğudur, bu politikalar yeteri kadar yüksek ya da başarılı tespit ve yakalama süreçleri ile birlikte kullanıldığı zaman, normu takip etme aksi stratejiye göre toplum içinde daha başarılı bir konuma gelir ya da durağan durum haline gelebilir. İkinci çıkarım ise, ilk çıkarımda bahsi geçen bütün pozitif durumların, sisteme af eklenmesi ile bozulabileceği ya da etkilerinin azalabiliceğidir. Bu durum sadece toplum yapısını bozmakla kalmayıp toplanan vergilerin etkisiz kullanımına da yol açacaktır. Simülasyonlarda görüldüğü üzere toplumun suça yönelmesi, aynı zamanda toplanan vergileri azaltacak, bu durum ise daha düşük tespit edilme ve tutuklanma oranlarına yol açacaktır.

Türkiye'nin geçmiş 100 yılında, 60'dan fazla af TBMM tarafından onaylanmıştır. Bunların sadece 9'u genel af niteliğindedir. Bahsi geçen bütün bu genel aflar çeşitli politik kaygılar veya temellere dayanmaktadır. Her ne kadar bu aflar, haksız yere hapis yatan nüfusun haklarını korumuş olsa da, genel nüfus üzerine etkisi düşünüldüğünden fazladır. Eğer amaç, geçmiş kanunların yol açtığı hataları düzeltmek ya da telafi etmek ise, genel af ve türevleri bu amaçta doğru araç olmayabilir. Daha başarılı ve etkin, tespit ve tutuklama politikaları böyle durumları daha yaşanmadan engelleyebilir ve masum toplumu suçlulara karşı koruyabilir.

Burada masum toplumun suça karşı korunmasından kasıt sadece mali anlamda değil, kişinin Heineke and Block (1975) çalışmasının önerdiği gibi suçlu konuma düşmenin yarattığı psikolojik maliyetten de korunmasını kapsamaktadır.

1 INTRODUCTION

Recidivism or relapsing into criminal behavior, often after the reception of sanctions or intervention for a previous crime is an important measure for the efficiency of the system of sanctions and pre-release policies. Data of reported recidivism rates in released prisoners from 25 countries suggests that in Denmark and Sweden, two countries where the re-conviction rates are the highest; re-conviction within 2 years of release is increased to 63% in 2013 from 29% in 2005 and from 43% in 2005 to 51% in 2011 respectively (Yukhnenko et al. 2019).

In Turkey, we see that the population of convicts in the penitentiary institution kept increasing except for the pandemic year where we see a 8.5% fall of convicts. Below, in Figure 1.1, we see the statistics for the period 2011-2020.

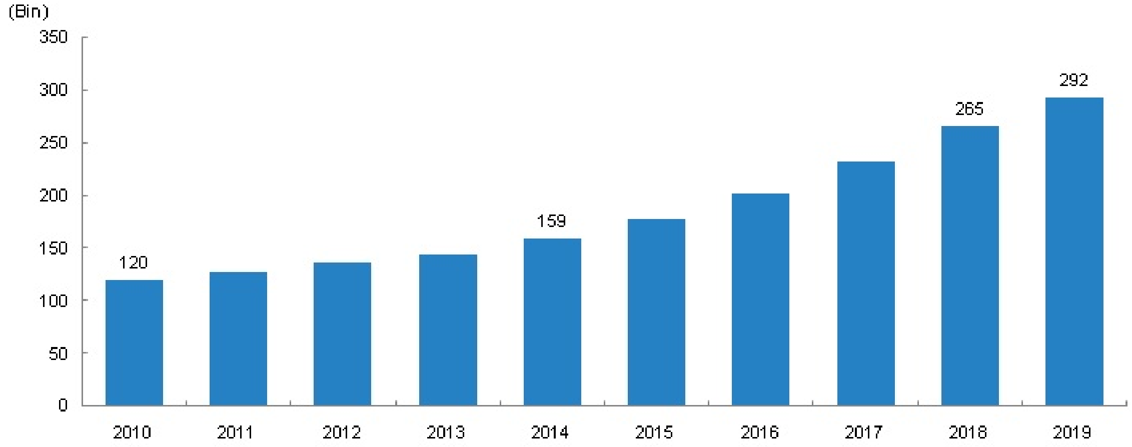


Figure 1.1 – Number of people in the penitentiary institution, 31 December, 2011-2020 (in thousands) TSI

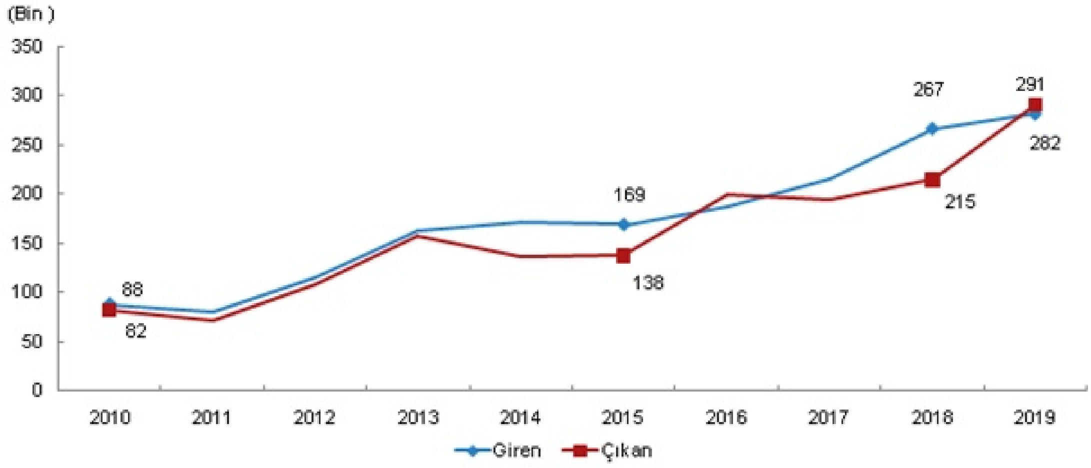


Figure 1.2 – Flow of people entering and leaving the penitentiary, 1 January-31 December, 2011-2020 (in thousands) TSI

1.1 Pre-release policies

Pre-release policies or acts of pardoning an offense can be grouped in three categories; amnesties, pardons and paroles. There might be different uses for these terms in different penal codes. Therefore we will try to explain these terms according to Turkish Penal Code (TCK-Türk Ceza Kanunu) for averting any ambiguity and misconception.

- We can find the description of ***amnesty*** in sub-article 65/1 of TCK: “Public action is dismissed in case of amnesty. The imposed punishments are abated together with all consequences.”. From this article, it can be deduced that after a declaration of an amnesty, not only all sentences are cancelled, also all criminal history is cleared too. We also need to note that the amnesty can not be granted to a group or person, instead it encapsulates a given crime. For the rest of this article we will take amnesty as an act of pre-release from prison.

- The following sub-article 65/2 describes the ***pardon***: “In case of pardon, the convict may be released from penitentiary where he is sentenced to imprisonment or the period of imprisonment is shortened or the imprisonment can be transformed to punitive fine.”. Also, we should add that the pardon can be declared for a group of person or a single person.

Therefore the main differences between the two mentioned acts of pre-release is that first, the amnesty is extended to persons who may not yet be convicted whereas a pardon is granted to convicted people and second, while the amnesty grants a clear criminal history to convicted, the pardon does not. This can have implications for recidivism as the convicted are judged according to the repetition of an offence. The article 58 states that “Provisions relating to recidivism are applied in case of commission of an offense after finalization of the decision for conviction. Execution of the sentence is not sought for adoption of this provision.”.

- While the article 107 of the Law on the Execution of Penalties and Security Measures no. 5275 describes the ***parole*** as the act of granting the partial extinguishment of criminal liability. The suspension of the sentence of a convict after serving the minimum term of the indeterminate penalty is considered for individual cases.

We would like to study the effect of such pre-release policies on crime rates regardless of their purpose whether they are granted for relieving the prison population, lowering institutional costs or holding an electoral promise. We would use a computational model based on a game theoretic model to analyse the impact of a amnesty and pardon on crime rate.

1.2 Definition of crime

Behaviours considered acceptable in a society are often enforced through sanctions when there is a conflict between self interest and social welfare. Deviant actions are punished and conformant behaviors are rewarded. Different forms of sanctions are used depending on the nature of the behaviours to be enforced from shame, disapproval to social discrimination and exclusion as well as penalties and fines. The creation and maintenance of norms are often accompanied by the creation of sanctions for enforcement. For example, law enforcement mechanisms and formal sanctions (fines and imprisonment) are designed by governments and organizations to create incentives to behave according to socially acceptable rules. However, sanctions are most of the time costly. The detection and punishment of deviant behaviours require resources.

Since foundation of TBMM in 1920, there have been over 60 pardons issued. Only nine of them can be counted as an amnesty, which first one issued in 1922 and the only amnesty issued by TBMM before Republic of Turkey’s foundation. Amnesty of 1922 granted amnesty to whom served 2/3 of their conviction periods and any convicted from occupied fields in Turkey. Following amnesties issued in 1923, 1933, 1950, 1960, 1963, 1966 1974 and 2000. Amnesty 1933 and 1973 had been issued due to being 10th and 50th year of foundation of Republic of Turkey, and Amnesty of 1960 issued right after TSK seized control of government. Amensty of 2000 named after Rahşan Ecevit who proposed the amnesty after hunger strikes all around penitentiaries in Turkey. Currenty there are expectations for another amnesty, since 2023 will be the 100th year of foundation of Republic of Turkey, and it's also supported by most of the political parties in Turkey.

It is also noted in Amnesty of 1938, along with other crimes, treachery also included in the law. Therefore 150 people who has been judged and found guilty by crime agains state by independence tribunals (İstiklal Mahkemeleri) pardoned from their crimes and their record had been cleaned. (TBMM Z.C., D. 5, C. 26, 29.6.1938) In 1950, another amnesty had been issued by TBMM, which in that time mostly consist of DP (Demokrat Parti). It is also remembered as first amnesty happened in multi-party system in Turkey. Although, this lawdidn't have "amnesty" in it's name, by its content it is qualified as an amnesty. Crimes like; murder, crimes against state property, communism, corruption, and embezzlement excluded from amnesty (TBMM Z.C., D. 9, C. 1, 14.7.1950). Extent of Amnesty of 1950, is smaller than previous ones, hence it's name "Law concerning pardon of certain crimes and punishments". The amnesties of 1960, 1963 and 1966 can be considered as succession of each other. Extend of these amnesties and Amnesty of 1974 is similiar to all other amnesties except 1938, which excluding more than 5 years of conviction time and only granted to certain crimes.

In this study, we do not attempt to answer questions about nature of justice and crime. We will use a simple definition of crime; “unjustified enrichment”. By enrichment, we do not only refer to monetary or instantaneous gains, we also attempt to include non-monetary gains. This decision can also be justified by the importance of these crimes in real life; as such crimes against direct wealth constitutes nearly 15% of crimes in Turkey in the period 2011-2020. Based on the most severe crime if the convicts entering the penitentiary commit more than one crime; among convicts entering the penitentiary institution between 1 January and 31 December 2019, 15.2% were guilty of theft, 12.4% injury, 7.0% manufacturing and trading drugs or stimulants, 5.4% acting against the Enforcement and Bankruptcy Law, 3.4% of murder. Also, many violent crimes are committed for financial benefits.

In this study, we focus on the role of sanctions to the motivation of cooperative behavior and investigate the effects of punishment and rehabilitation on recidivism.

The emergence and maintenance of cooperative action as a norm by self interested agents have been analysed extensively in politics, economics and sociology and using multiple tools. The game theoretic analysis investigates this problem mostly using ***prisoner's dilemma*** and ***hawk-dove games***. Agents can maintain the cooperative outcome only in repeated interactions and as one of the equilibria even when they are assumed to be perfectly rational. There are two problems with this analysis, the first is the multiplicity of equilibrium and the second is the hypothesis of perfect foresight and hindsight. Real life agents, on the other hand, mostly use trial-and-error inductive thinking, rather than the deductive rationale.

Here, we will use the hawk-dove game to capture the choice between prosocial (altruism and cooperation) and antisocial behavior (selfishness)[1]. This game represents the contest for a shareable resource. The contestants are hawks and doves representing aggressive and peaceful behaviour respectively. Doves share the resource equally in the absence of hawks, but when hawks are present, doves escape without fighting and leave the resource to hawks. When there are only hawks, they fight for the resource at a cost. The representation of two player hawk-dove game is given in Table 1.1 where v is the value of the contested resource, and c is the cost of an escalated fight. The best

		Player 2	
		H	D
Player 1	H	$\frac{v}{2}-c,\frac{v}{2}-c$	v,0
	D	0,v	$\frac{v}{2},\frac{v}{2}$

Table 1.1 – Hawk-Dove game

response against D is always H. However, the best response to H changes based on v and c:

- $\frac{v}{2}-c>0$ then the best response to H is H and the game is a prisoners' dilemma

[1] See Eldakar (2020) for a detailed analysis.

with $N(G) = \{(H, H)\}$.

- $\frac{v}{2} - c < 0$ then the best response to H is D and the game is a game of chicken with $N(G) = \{(H, D); (D, H)\}$.

There is negative externality as the resource is commonly available but substractable. The cooperative outcome (D,D) is never possible when the game is played once by rational agents under common knowledge. However, in real life we see that cooperation is possible. There are numerous studies investigating the circumstances under which cooperation is likely in these situations of negative externality. There are studies using evolutionary dynamics to capture both population aspect and rationality concerns.

As John Maynard Smith described in his book "Evolution and the Theory of Games", a hawk and dove game consists of two different species (H for hawks and D for doves) competing against shared resources[2]. The resource, like in his example, could be a favourable habitat, thereby obtaining it increases an individual's Darwinian fitness by a value v. It's not required for the individual that could not obtain the resource to have 0 fitness, therefore it can be seen as thievery in case of value of v. Though, John Maynard Smith suggests three types of strategies; display, escalate and retreat, to form a base game he assumed that there are only two types of strategy; escalate or retreat considering changing behaviour for an individual might be a complex manner. The payoffs are similar to Table 1.1.

Our goal here is to find a stable strategy, which is better than all other strategies, to understand where the population will evolve. John Maynard Smith assumed that all individuals will reproduce proportional to their Darwinian fitness. Hence the individual with more fitness will have a greater chance to breed. For a strategy to become stable it will need to yield more payoff than all other strategies, otherwise other strategies might invade the population. Let's assume there are two strategies in the population I and with a frequency of p and J. Outcomes to these two strategies as follows, which $E(I, J)$ denotes playing I against J.

[2]See also Hofbauer and Sigmund, 1998; Broom and Rychtář, 2013; McNamara and Leimar, 2020.

$$W(I) = W_o + (1-p)E(I,I) + pE(I,J)$$
$$W(J) = W_o + (1-p)E(J,I) + pE(J,J)$$

Suppose that I is the stable one, which requires it to yield more fitness than the other one, implying $W(I) > W(J)$. In this case either $E(I,I) > E(I,J)$ or $E(I,I) = E(I,J)$ and $E(I,J) > E(J,J)$. As John Maynard Smith states any strategy that satisfies preceding conditions can be qualified as a stable strategy.

For our basic game, it can be deduced that H is an *evolutionarily stable strategy* ESS, so long $v > c$ and it's not allowed to play mixed strategies as we stated before. In the case of $v < c$, there are no ESS for the game. Therefore a mixed solution is required. Since it's required to satisfy $E(H,I) > E(D,I)$, we can write a mixed strategy with playing Hawk with a probability of p.

$$\begin{aligned} pE(H,H) + (1-p)E(H,D) = pE(D,H) + (1-p)E(D,D) \\ \frac{1}{2}(v-c)p + v(1-p) = \frac{1}{2}v(1-p) \\ p = \frac{v}{c}. \end{aligned}$$

Therefore, the proportion between v and c determines the probability. It is also required to note that we have made three assumptions in the model; an infinite random-mixing population, asexual reproduction and symmetrical contest. Later Chen et al. (2017) proposed a generalization of the Hawk-Dove Game for an arbitrary number of agents and found the stability conditions for the coexistence of both species.

Sakiyama (2021) extends the setup using spatial game theory and network systems. This setup allows the interaction with neighbours and a more realistic environment. The evolution of the hawk-dove game was analyzed using the formation and disappearance of links between agents in the population. The cooperation may appear as a dominant strategy in these types of environment.

Another extension by Krivan and Cressman (2022) is about the asymmetry of costs. This asymmetry relates to the fact that hawk is a loser or a winner. This model predicts three evolutionary outcomes where only hawk strategy is used in the population; both hawk and dove strategies survive or hawk and a mixed strategy. This result is in contrast to the classic symmetric hawk-dove game where hawk and dove strategies survive or hawk and a mixed strategy can not be evolutionarily stable.

2 LITERATURE REVIEW

Economics as the study of how to allocate scarce resources to meet different needs has entwined with criminal law in the context of the punishment of criminal acts. There are two approaches to the study of criminal law in economics: normative and positive. The normative approach concerns with the optimal regulation of socially undesirable behaviour. The positive approach, on the other hand, deals with the investigation of a rational to existing criminal law enforcement process (Hylton, 2019).

2.1 Choice of Criminal Activity

The normative studies go back to eighteenth century. Bentham (1781) suggests that punishment is "an artificial consequence annexed by political authority to an offensive act" to completely deter this offensive conduct. The complete deterrence of the conduct is achieved when the level of criminal penalty eliminates the prospect of gain[3]. The punishment aims at increasing the happiness of the society. Since the punishment mechanism is based on the infliction of pain, it is a mischief as well and "it ought only to be admitted in as far as it promises to exclude some greater evil." In his work, Bentham gives the clue to the definition, the extent and modes of punishment. The deterrence of criminal acts have been defined as punishment by reformation, disablement and compensation and deterrence.

These ideas have been theorized by Becker (1968) which inquired "how many resources and how much punishment should be used to enforce different kinds of legislation?" and "how many offenders should be permitted and how many offenders should go unpunished? with the purpose of finding an optimal balance between cost of crimes

[3] This idea goes back to Beccaria (1764). However, Bentham (1781) proposed a more detailed analysis.

and expenditures to prevent and/or punish crimes by using a static model to explain individual's crime preferences. An individual decides whether to commit a crime or not by calculating costs and benefits from the crime in a certain period of time, in other words considers "if a crime pays or does not pay" while the society can decide whether to increase the probability of discovering and apprehending by increasing expenditures on law enforcement and judicial system or to increase the punishment of those convicted to deter potential offenders. Becker (1968) suggested that an increase in the probability of being convicted or the punishment when found guilty can decrease the number of crimes committed as the opportunity cost of a criminal act increases without any increase in benefits.

The prevention of criminal activities in this static model is based on the deterrent effect of the severity of the punishment. From this perspective, fines are just the price of illegal activities. Therefore it has a deterrence effect on people who can not afford a crime, which in turn creates an environment for rich people to commit a crime.

Ehrlich (1973) expands Becker's work by introducing the allocation of time over legitimate and illegal activities rather than describing them as mutually exclusive activities; then, by defining both punishment and reward from illegal and legitimate activities instead of just focusing on the cost of punishment and third, by measuring the incarceration effect. This theory has been tested using US data on index crimes. Based on these studies, Heineke and Block (1975) introduces the psychological frame of criminal activities to the previous framework by encapsulating non-monetary effects of committing a crime. Agents can have preference for honesty, or preference for illegal activities. The results of Becker (1968) and Ehrlich (1973) are equivalent when these activities are monetarily equivalent and independent of the level of wealth.

There are other similarities between the models mentioned above. First, all models are static by their nature. Time perception in these models is absent hence it becomes impossible to capture the effect of punishment of criminal activities on subsequent decisions. Second, they all consider penalties and fines as methods of punishment of criminal conduct, thus the effect of incarceration becomes redundant.

Block and Lind (1975) do not specifically try to determine the optimal level of sanctions but focus on the probability of punishment and argue that under strong assumptions about the function of utility. The study attempts to capture different kinds of deterrence and incentive with respect to legal and criminal wealth gain, cost of the crime, return of the crime in case of conviction and last but not least the probability of apprehension and period of sentence and finds out any proportional change in probability of apprehension has a greater deterrence effect than same proportional change in severity of punishment. Later, Schmidt and Witte (1984) and Witte (1980) proposed similar models with ambiguous results regarding the variables of concern and stressed the importance of empirical analysis for appropriate policy implications[4].

More recent studies analyse the effect of incarceration (Levitt 1996, Drago et al. 2009), parole and bail institutions (Kuziemko 2007b, Kuziemko 2007a), the effect of incarceration on the supply of crime in the economy (Freeman 1996, 1999), and optimal law enforcement (Polinsky and Shavell, 2000). For instance, Drago et al. (2009) studies the effect of sentence times on the recidivism. based on the empirical evidence of the Collective Clemency Bill passed by the Italian Parliament in 2006 which is a pardon, not an amnesty by our terms. They show that increasing sentence times by 50% will result a roughly 35% reduce in recidivism. Results also reveals a certain dilemma, in the situation of a pardon; while granting a pardon would decrease recidivism rates since it has a deterrence effect because of adding pardoned sentence to new sentence time, it also decreases expected sentence time. Therefore policy makers should know elasticities of these policies and pass such an act based on this information.

Further analysis by Calabresi and Melamed (1972), Hylton (2005) and Curry and Doyle (2016) suggest models categorizing legal rules. Calabresi and Melamed (1972) categorises these rules into two; liability rules and property rules. While liability rules do not aim to complete deterrence, property rules does. Also property rules have two categories differing in their transaction costs. Hylton (2005), suggests if maximum offender gain exceeds minimum victim harm and transaction is costly, system should apply liability rule and internalize the harm. All other combinations of these two conditions falls under the property rule, which implies a complete deterrence.

[4]For empirical studies on the impact of crime deterrence policies and economic conditions on crime, see Corman and Mocan (2000), Tauchen et al. (1994). Corman and Mocan (2000) claims that any increase in deterrence variables should also decrease criminal activity and finds the deterrence effect of arrests and police force using a model based on Ehrlich (1973).

These models were static in their nature, by definition static models are snapshots of a specific time therefore agents in such models account for utility or welfare at a given time, or a period. But usually, the decision to take a criminal action is affected by the past experiences of criminal action. It is not possible to study recidivism with two-period models. Leung (1995) formulate and solve a general dynamic deterrence model with recidivistic behaviour and compares the impact of the likelihood of punishment and the severity of punishment. It is found that the certainty of punishment is more effective in deterring crime than the severity of punishment.

Freeman (1996) tries to explain reasons of crime in USA and the deterrence effect using the crime rate in non-institutionalized population. Though prison population increases in 1977 to 1992 in USA, the crime rate in non-institutionalized population also increases by 163%. This empirical evidence brings forth the following questions: ***Do juridical system opens more jobs in criminal market by incarcerating the criminals? If incarceration policies are not sufficient to deter the criminals from criminal activities, do those policies creates a vicious cycle?*** There is also evidence form labour market from 1989 Boston Youth Survey suggesting that compared with $10.00 per hour that a criminal can earn young people can earn only $7.50 per hour from legitimate work. Together with the data on public spending suggesting that in 1995 California spent more on prisons than on higher education, increased legitimate opportunities for less skilled workers in young population is suggested, among with other policies, to prevent further crime rates. Burdett et al. (2003) provides a dynamic model of crime using search theory to explain relationship between crime, unemployment and inequality. Mocan et al. (2005) proposes a dynamic model of criminal activity by introducing legal and criminal human capital which affect the potential income in the relevant sector. The participation in legal and criminal sectors increases the human capital for each whereas investment can increase legal human capital as well. In this setup, their model introduces recessions, imprisonment/rehabilitation scenarios, sanctions and returns to human capital to analyse their impact on crime.

Moreover, it is important to account for individual heterogeneity by acknowledging that previous experience of criminal activity can change the rewards as well as sanctions thereof and thus affect the response to different deterrence and pre-release policies (Wilson, 1994). Here, we will use population dynamics to allow for different behaviours and backgrounds in the population and analyse the dynamics of these behaviours.

2.2 Population Dynamics

In economics (externalities, macroeconomic spillovers, central markets); in biology (animal conflict, genetic natural selection); in logistics (traffic, choice of method of transportation); in computing (selfish routing), we see that models of collective behavior are derived from clear micro-foundations. These population games present the strategic interaction between large numbers of individuals where each individual is a small unit with limited number of roles and interacts anonymously. In these cases, it may be possible to arrive at a theory, starting with an explanation of how individuals update their strategies and behaviors, that is, by defining a revision protocol and showing how behavior changes in general (Sandholm, 2010).

The basic assumptions of game theory, which provides a systematic analysis of the strategic interaction between agents, are rationality and perfect knowledge. Agents act with the purpose of maximizing their self-interest under perfect knowledge. Traditionally, predictions made with game theory are based on equilibrium concepts. The concept of equilibrium, on the other hand, is based on the assumption of the knowledge of this equilibrium. In such an environment, the problems we encounter in general are which equilibrium to choose when there are many equilibria, the realism of the concept of the hyperrational individual, and the lack of a dynamic aspect of the environment. The assumption of coordination of beliefs to achieve a certain equilibrium is difficult to defend when many agents interact.

The first answer to these questions was presented by Maynard Smith (1982) with the concept of evolutionary stability. Evolutionary stability aims to improve the Nash equilibrium to obtain a better estimate. Although a dynamic process is not defined, local stationarity analysis checks whether the equilibrium is re-established with minor disturbances.

An explicit dynamic modeling of individual decisions is another alternative. Dynamic modeling of behavior in population games has been studied extensively as it allows the description of strategic interaction in many societal contexts by relaxing the assumptions of rationality and common knowledge. Evolutionary game theory, which models strategic interaction with weaker rationality criteria, presents such a dynamic process. In evolutionary modelling, agents are supposed to change strategies occasionally and using myopic decision rules. This type of modelling is well suited for the study of the behaviour of large, strategically interacting populations.

This study will use imitation dynamics to account for the revision of behaviours and attitudes. The choice of criminal activity is based on the performance of this activity compared with other alternatives. As we are studying a strategic environment, imitation dynamics is useful to model the change in behaviours and attitudes (Weibull, 1995; Björnerstedt and Weibull, 1996; Hofbauer and Sigmund, 2003). Then, we will study the asymptotic behavior of these dynamics[5].

[5]For results on asymptotic stability, see Nachbar (1990), Hofbauer (2000) and Sandholm (2001, 2010).

3 THEORETICAL MODEL

3.1 Population

Most analyses in game theory is based upon the concept of rationality. Rationality is preference consistency in the sense that agents choose best responses in strategic interactions. However, rationality usually can not help us predict the human behaviour in real life situations. However, Maynard Smith and Price (1973) have shown that game theory can be applied to the behavior of animals and introduced the concept of evolutionary stable strategies. Evolutionary game theory is based on the Darwinian notion of the survival of the fittest in the sense that good strategies can diffuse in the population rather than being learned by rational agents.

Instead of agents choosing particular strategies from a set of strategies, actions or behaviours now we have each agent representing a certain strategy or behaviour. In biology, species have genotypic variants and one or another genotypic variant is inherited or mutates. This can be used to treat evolution of behaviours in human society, where society has a set of alternative behavioural traits and individuals inherit or choose among them.

In our context, there are two possible behaviour to adopt: behaving Hawkish (H) or Dovish (D). An individual may be act cooperatively for the benefit of the society (D) or act selfishly at the expense of the society (H). We accept prosocial cooperative behaviour as the norm of the society. In this sense, acting selfishly at the expense of the society will constitute a crime that can be punished with a sanction upon detection. The population consists initially of individuals of all types of behaviour i.e. there are agents acting cooperatively and others acting selfishly.

We consider a society of Hawks (H) and Doves (D) N_t at time t. The society encourages cooperative attitude and enforces the selection of behaviour D. The selection of behaviour H is considered a crime. The aggressive individual is convicted and imprisoned if this criminal behavior is caught with a probability of δ. The population of individuals is made of a criminal group C_t and an innocent group I_t where the population of criminals are either convicted or not convicted. The share of criminals in the population is $h_t = \frac{C_t}{N_t}$, thus the share of innocent population is $1 - h_t$.

At the beginning of each $t = 0, 1, 2, ...$, the individual decides whether to belong to the criminal population (behavior H) or to **completely leave the criminal population and never commit a crime in the future** (behavior D). This choice is made based on imitation. If being innocent is better performing, the criminal imitates this behavior and changes its strategy accordingly.

Agents following the norms of the society, have to pay a fee to keep the legal system working to catch and punish criminal activity. This fee ϕ works just like a proportional tax on the payoff of agents. The budget of legal system includes the costs of legal system and imprisonment and the revenues collected by the fees from agents $\sum_{i=1}^{I_t} u_i^D \phi = I_t u_i^D \phi$. This fee affects the probability of criminals to be caught and get convicted as follows:

$$\begin{aligned}\delta_t &= p(\frac{u_i^D I_t \phi}{N_t}) \\ &= p((1 - h_t) u_i^D \phi).\end{aligned}$$

This can be conceived as per capita cost of detection and imprisonment. We suppose that there are decreasing returns to scale in detection process and therefore the probability of detection increases with collected fees but at a decreasing rate. We will use the following function for the probability of detection to represent this modelisation:

$$\delta_t = \epsilon \sqrt{(1 - h_t) u_i^D \phi}.$$

When a criminal is caught, he will be imprisoned forever.

We will consider three cases to study the impact of amnesty on the criminal activity.

- First, we sill study the economic environment without amnesty.
- Second, we will introduce the amnesty. In case of amnesty, with a probability θ, there will be remissions. The difference between pardon and amnesty is that in case of amnesty, the criminal record is deleted when the agent goes out of prison and in case of pardon, there is a share of imprisoned agents joining back the conventional population. This share can be constant or determined by a certain rule. We will investigate the impact of amnesty by comparing to the no amnesty case.
- Third, we will analyze the pardon with an agent-based model. In that case, we can add the history of crimes and the possibility of being granted a remission or exemption if the criminal has been caught less than a tolerable amount of time R.

3.2 Interaction

The agents are randomly paired each time $t = 0, 1, 2, ...$ and play the hawk-dove game that is given in Table 1.1 and earn the payoffs associated with their composition of behaviours. At the end of the period, with a probability of δ, non-cooperative agents are convicted and imprisoned. Their payoff from that period during their entire imprisonment is 0.

Lastly, innocent agents may randomly see a crime happening and report the criminals or they may be vigilante. If we want to find a connection between remissions and demand for justice, we have define this behaviour related to other variables of the model. Now, we will define the expected utility of each agent of different behavioural traits.

3.3 No amnesty

First, we will start with the case of no amnesty. After being caught, criminals will be sentenced forever.

3.3.1 Payoffs

The expected utility of an innocent agent is

$$u_t^D = (1-\phi)\frac{I_t}{N_t}\frac{v}{2}. \tag{1}$$

and the expected utility of a criminal agent is

$$\begin{aligned} u_t^H &= (1-\delta_t)\left(\frac{I_t}{N_t}v + \frac{(1-\delta_t)C_t}{N_t}(\frac{v}{2}-c)\right) \\ &= (1-\epsilon\sqrt{(1-h_t)u_i^D\phi})\left(\frac{I_t}{N_t}v + \frac{(1-\epsilon\sqrt{(1-h_t)u_i^D\phi})C_t}{N_t}(\frac{v}{2}-c)\right) \end{aligned} \tag{2}$$

Then, the average utility in the population will be

$$\bar{u}_t = \frac{C_t}{N_t}u_t^H + \frac{I_t}{N_t}u_t^D \tag{3}$$

3.3.2 Population dynamics

In traditional game theory, rational agents choose their strategies by calculating their best responses based on their complete and transitive preference orders. We can relax the rationality assumption by allowing agents initially to be assigned to actions or behaviours and to change these actions or behaviours occasionally using myopic decision rules. This type of modelling is well suited for the study of the behaviour of large, strategically interacting populations.

We will use imitation dynamics to account for the revision of behaviours and attitudes. The choice of criminal activity is based on the performance of this activity compared with other alternatives. The share of criminals in the population is $h_t = \frac{C_t}{N_t}$, thus the share of innocent population is $1-h_t$. The change in the share of criminals is given by

$$h_{t+1} = h_t + \beta h_t(u_t^H - \bar{u}_t) = h_t + \beta h_t \left(u_t^H - (h_t u_t^H + (1-h_t)u_t^D)\right)$$
$$= h_t + \beta h_t(1-h_t)(u_t^H - u_t^D) \tag{4}$$
$$= h_t + \beta h_t(1-h_t)\left((1-\delta_t)\left((1-h_t)v + (1-\delta_t)h_t(\frac{v}{2}-c)\right) - (1-\phi)(1-h_t)\frac{v}{2}\right)$$

The continuous time version of the previous dynamics is given by the following function:

$$\frac{dh}{dt} = \dot{h} = \beta h(1-h)\left((1-\delta)\left((1-h)v + (1-\delta)h(\frac{v}{2}-c)\right) - (1-\phi)(1-h)\frac{v}{2}\right)$$
$$= \beta h(1-h)\left((1-h)\frac{1-2\delta+\phi}{2}v + (1-\delta)^2 h(\frac{v}{2}-c)\right) \tag{5}$$

3.3.3 Steady state distribution of behaviours

Now, we will study the asymptotic properties of the above dynamics. The behaviours in the population change through imitation dynamics. It is important to study the rest points of this dynamic system as once the population reaches this equilibrium state, the share of behaviours or attitudes in the population will not change. There can be two types of steady states: i. *pure state* where the entire population has the same behaviour and ii. *mixed state* where a mixture of behaviours is observed. The steady state is reached when $\dot{h} = 0$.

$$\dot{h} = \beta h(1-h)\left((1-h)\frac{1-2\delta+\phi}{2}v + (1-\delta)^2 h(\frac{v}{2}-c)\right) = 0 \tag{6}$$

Equation 6 has the trivial pure equilibria $h^* = 0$ and $h^* = 1$. If the population reaches the first there are only innocent agents while if it reaches the second only criminals remain. The stability of these pure states is a function of payoffs of the game as well as detection probability thus the cost of imprisonment and detection. By stability, we mean the asymptotic stability (Liapunov stability)[6].

[6]For further reference, see Hirsch and Smale (1974).

Let $F(h) = (1-h)\frac{1-2\delta+\phi}{2}v + (1-\delta)^2 h(\frac{v}{2} - c)$. The value of this function at each pure state will reveal its stability. At $h^* = 0$, we have,

$$F(0) = \frac{v}{2}\left(1 - \epsilon\sqrt{2(1-\phi)v\phi} + \phi\right).$$

For $h^* = 0$ to be stable, $F(0)$ must be negative. Then

$$\begin{aligned} 1 - \epsilon\sqrt{2(1-\phi)v\phi} + \phi < 0 \\ \left(\frac{1+\phi}{\epsilon}\right)^2 \frac{1}{2(1-\phi)\phi} < v \\ 1 + 2(1-\epsilon^2 v)\phi + (1+2\epsilon^2 v)\phi^2 < 0 \end{aligned}$$

The last expression is a quadratic function of ϕ. The coefficient of ϕ^2 is positive. The discriminant $4(\epsilon^2 v - 4)\epsilon^2 v$ is non-negative if $\epsilon^2 v - 4 > 0$ or $v < \frac{4}{\epsilon^2}$. If the discriminant is negative, $F(0) > 0$ as the coefficient of ϕ^2 is positive; if the discriminant is positive then $F(0) < 0$ for the values of ϕ in the interval between the roots of this quadratic equation. The roots are $\frac{-1+\epsilon^2 v+2\epsilon\sqrt{(\epsilon^2 v-4)v}}{1+2\epsilon^2 v}$ and $\frac{-1+\epsilon^2 v-2\epsilon\sqrt{(\epsilon^2 v-4)v}}{1+2\epsilon^2 v}$. The first root is positive as $v < \frac{4}{\epsilon^2}$ should be satisfied so that the discriminant is positive.

- The second root is positive if $0 < 1 + 14\epsilon^2 v - 3\epsilon^4 v^2$, therefore for values of $4 < \epsilon^2 v < \frac{7+2\sqrt{13}}{3}$, both roots are positive and for values of the contribution ϕ in between these roots $h^* = 0$ is stable. from the income in the unit interval, $F(0) > 0$. This rest point is unstable.

- The second root is negative if $0 > 1 + 14\epsilon^2 v - 3\epsilon^4 v^2$, therefore for values of $\frac{7+2\sqrt{13}}{3} < \epsilon^2 v$, for values of the contribution ϕ between 0 and the first root, $h^* = 0$ is stable. This rest point is otherwise unstable.

At $h^* = 1$, we have,

$$F(1) = \frac{v}{2} - c.$$

For $h^* = 1$ to be stable $F(1)$ must be positive i.e. $\frac{v}{2} - c > 0$ implying $v > 2c$. If the value of the resource is high enough then the rest point is stable.

Proposition 1. *All agents behaving against norms of the society can be stable if the value of the resource is high enough i.e.* $v > 2c$*. All agents complying with the norms of the society can also be stable. The stability condition is described above and based on a sufficiently high value of* $\epsilon^2 v$*.* ϵ *represents the efficiency of the detection process, so if detection process is efficient enough then all individuals complying to the norms can survive the evolution.*

Now, we will analyse the other non trivial rest points of the imitation dynamics. In other words, we will find the solutions of the equation $F(h^*) = 0$.

$$\begin{aligned}
&(1-h)\frac{1-2\delta+\phi}{2}v + (1-\delta)^2 h(\frac{v}{2}-c) = 0\\
&(1-h)\frac{1-2\epsilon\sqrt{(1-h)(1-\phi)(1-h)\frac{v}{2}\phi}+\phi}{2}v\\
&+\left(1-\epsilon\sqrt{(1-h)(1-\phi)(1-h)\frac{v}{2}\phi}\right)^2 h(\frac{v}{2}-c) = 0
\end{aligned} \tag{7}$$

This equation is rather complex and for a better illustration we will use some configurations:

- $v = 2c$ and $\phi = 0.02$: For these values, equation 7 admits 3 solutions and none is eligible given that h^* should be in the unit interval. Figure 3.1 represents these solutions.

- $v = c$ and $\phi = 0.02$: For these values, equation 7 admits 6 solutions. Figure 3.2 represents these solutions.

- $v = 3c$ and $\phi = 0.02$: For these values, equation 7 admits 6 solutions and Figure 3.3 represents these solutions.

Now, we can check for the stability of the mixed state at each configuration. For the first configuration, there is only one root in the unit interval. Consequently, if $F(1) < 0$ and $F(0) > 0$ the mixed state will be stable.

- $v = 2c$ and $\phi = 0.02$: For the first configuration, there is only one root in the unit interval. Consequently, if $F(1) < 0$ and $F(0) > 0$ at this configuration, the mixed state will be stable. Although $F(0)$ can be negative for higher values of

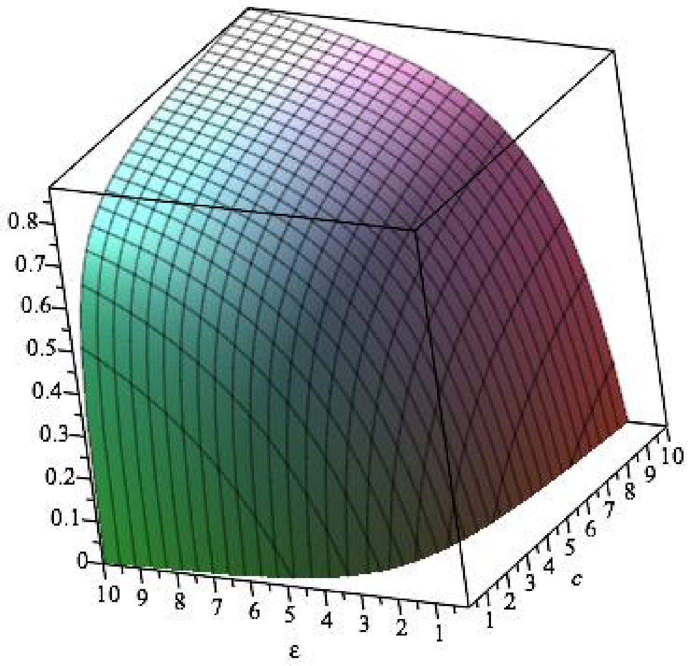


Figure 3.1 – Mixed states for $v = 2c$ and $\phi = 0.02$

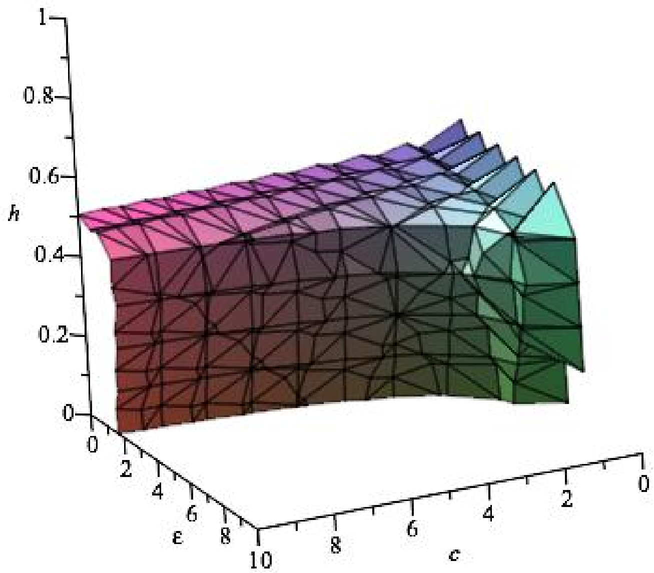


Figure 3.2 – Mixed states for $v = c$ and $\phi = 0.02$

$\epsilon^2 c$, $F(1) > 0$. These mixed states are unstable.

- $v = c$ and $\phi = 0.02$: For this configuration, there are at most 6 solutions. $F(0)$

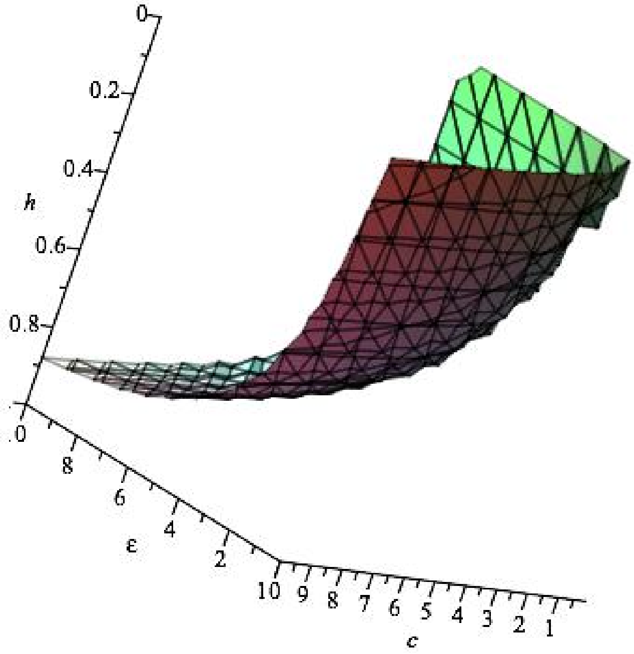


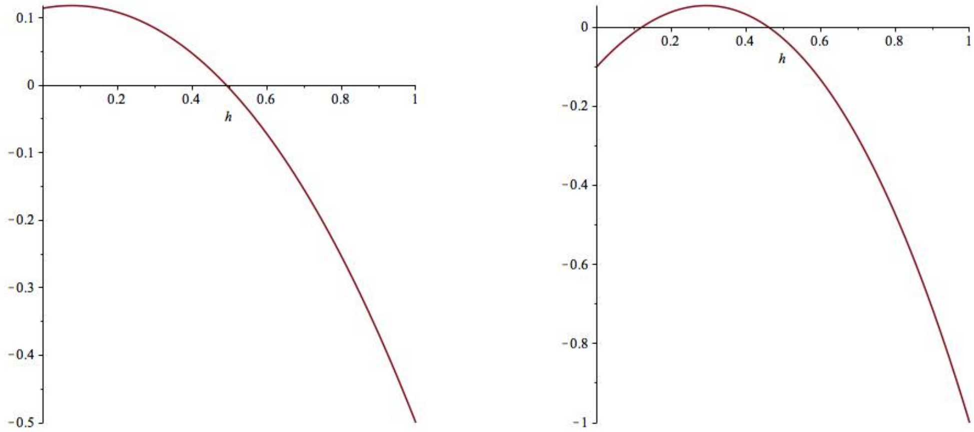
Figure 3.3 – Mixed states for $v = 3c$ and $\phi = 0.02$

can be negative for higher values of $\epsilon^2 c$ and $F(1) < 0$. In case there is one root, this will be definitely be stable. For $\epsilon = 4$ and $c = 1$, there is one root (0.49) and in Figure 3.4(a), we see that this rest point is stable. In case of two roots, one will be unstable and the other will be stable. Figure 3.4(b) illustrates the case of two rest points using $\epsilon = 4$ and $c = 2$. This is the graph of $F(h)$ and according to its sign around rest points, the first 0.12 is unstable while the second (0.46) is stable.

- $v = 3c$ and $\phi = 0.02$: For this configuration, there are at most 6 solutions. $F(0)$ can be negative for lower values of $\epsilon^2 c$ and $F(1) > 0$. If there is one rest point, this rest point will be unstable. Figure 3.3 shows that mostly the roots in the unit interval are in the area where both ϵ and c are high. We can use the above specification for ϵ and c for the illustration of an unstable rest point. For $\epsilon = 4$ and $c = 1$, there is one rest point (0.23) in the unit interval and in Figure 3.5(a), we see that this rest point is not stable. In case of high cost, Figure 3.5(b) illustrates the case of one rest point using $\epsilon = 4$ and $c = 2$. This is the graph of $F(h)$ and according to its sign around the rest point, the rest point 0.45 is unstable.

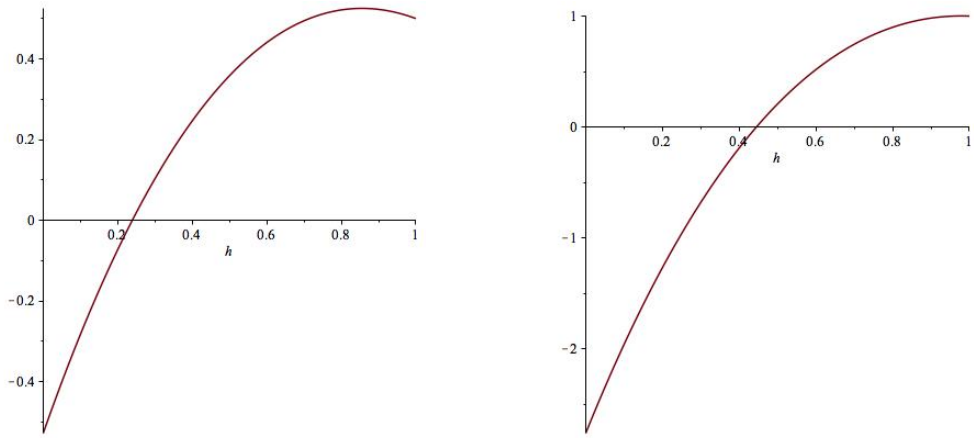
We see that a large enough resource to cover the cost of confrontation or more

Figure 3.4 – Stability of mixed states for $v = c$, $\epsilon = 4$ and $\phi = 0.02$



(a) $F(h)$ for low level of cost $c = 1$ (b) $F(h)$ for high level of cost $c = 2$

Figure 3.5 – Stability of mixed states for $v = 3c$, $\epsilon = 4$ and $\phi = 0.02$



(a) $F(h)$ for low level of cost $c = 1$ (b) $F(h)$ for high level of cost $c = 2$

leads to a stable state of only criminal population. There can be stable states otherwise. The following proposition summarises the results.

Proposition 2. *The economic environment may have stable rest points according to first the amount of resources to be shared compared with the cost in case criminals confront each other and the efficiency of detection process.*

- *In case the resource is large enough to only cover the cost of confrontation, there is only one mixed state and it is unstable.*
- *In case the resource can not cover the cost of confrontation, there can be several*

mixed states depending on the level of the cost and the efficiency and some may be stable.

- *In case the resource is larger than the level to cover the cost of confrontation, the mixed state is unstable. and it is unstable.*

3.4 Amnesty

Now, we will define the expected utility of each agent of different behavioural traits in case of amnesty. The criminals who are caught will be sentenced but randomly, there will be amnesties issued for the whole criminal population. We suppose that the probability of an amnesty being issued is α.

3.4.1 Payoffs

The expected utility of an innocent agent is

$$u_t^D = (1-\phi)\frac{I_t}{N_t}\frac{v}{2}. \tag{8}$$

and the expected utility of a criminal agent is

$$\begin{aligned} u_t^H &= (1-(1-\alpha)\delta_t)\left(\frac{I_t}{N_t}v + \frac{(1-(1-\alpha)\delta_t)C_t}{N_t}(\frac{v}{2}-c)\right) \qquad (9) \\ &= (1-(1-\alpha)\epsilon\sqrt{(1-h_t)u_i^D\phi})\left(\frac{I_t}{N_t}v + \frac{(1-(1-\alpha)\epsilon\sqrt{(1-h_t)u_i^D\phi})C_t}{N_t}(\frac{v}{2}-c)\right) \end{aligned}$$

Then, the average utility in the population will be

$$\bar{u}_t = \frac{C_t}{N_t}u_t^H + \frac{I_t}{N_t}u_t^D \tag{10}$$

3.4.2 Population dynamics

We will use again imitation dynamics to account for the revision of behaviours and attitudes. The change in the share of criminals is given by

$$
\begin{aligned}
h_{t+1} &= h_t + \beta h_t(u_t^H - \bar{u}_t) = h_t + \beta h_t \left(u_t^H - (h_t u_t^H + (1-h_t)u_t^D)\right) \\
&= h_t + \beta h_t(1-h_t)(u_t^H - u_t^D) \\
&= h_t + \beta h_t(1-h_t)\left[(1-(1-\alpha)\delta_t)\left[(1-h_t)v + \right.\right. \\
&(1-(1-\alpha)\delta_t)h_t(\frac{v}{2} - c)\Big] - (1-\phi)(1-h_t)\frac{v}{2}\Big]
\end{aligned} \tag{11}
$$

The continuous time version of the previous dynamics is given by the following function:

$$
\begin{aligned}
\frac{dh}{dt} = \dot{h} =& \beta h(1-h)\Big((1-(1-\alpha)\delta)\Big((1-h)v + (1-(1-\alpha)\delta)h(\frac{v}{2} - c)\Big) \\
&-(1-\phi)(1-h)\frac{v}{2}\Big) \\
=& \beta h(1-h)\left((1-h)\frac{1-2(1-\alpha)\delta+\phi}{2}v + (1-\delta)^2 h(\frac{v}{2} - c)\right)
\end{aligned} \tag{12}
$$

3.4.3 Steady state distribution of behaviours

The study the asymptotic properties of the above dynamics under amnesty will allow us to compare the stability of cooperative outcome based on sanction and amnesty policies. The steady state is reached when $\dot{h} = 0$.

$$
\dot{h} = \beta h(1-h)\left((1-h)\frac{1-2(1-\alpha)\delta+\phi}{2}v + (1-(1-\alpha)\delta)^2 h(\frac{v}{2} - c)\right) = 0 \tag{13}
$$

Equation 13 has the trivial pure equilibria $h^* = 0$ and $h^* = 1$. The stability of these pure states is now a function of payoffs of the game as well as detection and amnesty probability. Let $G(h) = (1-h)\frac{1-2(1-\alpha)\delta+\phi}{2}v + (1-(1-\alpha)\delta)^2 h(\frac{v}{2} - c)$. The value of this function at each pure state will reveal its stability. At $h^* = 0$, we have,

$$
G(0) = \frac{1-2(1-\alpha)\epsilon\sqrt{(1-\phi)\frac{v}{2}\phi} + \phi}{2}v.
$$

For $h^* = 0$ to be stable $G(0)$ must be negative. Then

$$\frac{1 - 2(1-\alpha)\epsilon\sqrt{(1-\phi)\frac{v}{2}\phi} + \phi}{2} v < 0$$
$$\left(\frac{1+\phi}{2(1-\alpha)\epsilon}\right)^2 \frac{2}{(1-\phi)\phi} < v$$
$$\left(\frac{1+\phi}{(1-\alpha)\epsilon}\right)^2 < 2(1-\phi)\phi v$$
$$1 + (1 + 2v(1-\alpha)^2\epsilon^2)\phi^2 + (2 - 2v(1-\alpha)^2\epsilon^2)\phi < 0$$

The last expression is a quadratic function of ϕ. The coefficient of ϕ^2 is positive. The discriminant $4v\epsilon^2(-1+\alpha)^2(v\epsilon^2(-1+\alpha)^2 - 4) < 0$ if $v < \frac{4}{\epsilon^2(-1+\alpha)^2}$. As the coefficient of ϕ is positive, this means $G(0) > 0$ for values of v in that interval. This rest point is unstable. If $v > \frac{4}{\epsilon^2(-1+\alpha)^2}$ then there will be two roots to the quadratic equation and depending on whether the roots are in the unit interval the stability can be evaluated.

At $h^* = 1$, we have,

$$G(1) = \frac{v}{2} - c.$$

For $h* = 1$ to be stable $G(1)$ must be positive. $\frac{v}{2} - c > 0$ if $v > 2c$. If the value of the resource is high enough then the rest point is stable.

Proposition 3. *Only one of the pure states is stable under the current setting. The state where all agents behave against the social norm is stable if the value of the resource is high enough i.e.* $v > 2c$. *The survival of compliance to norms by all is weakened by even a small chance of amnesty.*

For the non trivial rest points of the imitation dynamics, we will find the solutions of the equation $G(h^*) = 0$.

$$
\begin{aligned}
&(1-h)\frac{1-2(1-\alpha)\delta+\phi}{2}v+(1-(1-\alpha)\delta)^2h(\frac{v}{2}-c)=0\\
&(1-h)\frac{1-2(1-\alpha)\epsilon\sqrt{(1-h)(1-\phi)(1-h)\frac{v}{2}\phi}+\phi}{2}v \\
&+(1-(1-\alpha)\epsilon\sqrt{(1-h)(1-\phi)(1-h)\frac{v}{2}\phi})^2h(\frac{v}{2}-c)=0
\end{aligned} \tag{14}
$$

Now that we have another variable, we will use a variety of levels of amnesty probability to illustrate low ($\alpha = 0.1$), medium ($\alpha = 0.3$) and high ($\alpha = 0.5$) amnesty probabilities.

- $v = 2c$ and $\phi = 0.02$: For these values, equation 14 admits 3 solutions and none is eligible given that h^* should be in the unit interval. Figure 3.6 represents these solutions for the above level of α. We see that as α increases, the mixed state level of criminals also increases at each (ϵ, c) pair.

- $v = c$ and $\phi = 0.02$: For these values, equation 14 admits 6 solutions. Figure 3.7 represents the solutions that are in the unit interval for each level of amnesty probability. We see that in this case as the level of resource fall below the cost of confrontation, the mixed state share of criminals is lower at each (ϵ, c) pair.

- $v = 3c$ and $\phi = 0.02$: For these values, equation 14 admits 6 solutions and Figure 3.8 represents these solutions. Compared with the first configuration when we increase the value of the resource then the mixed state is higher at each (ϵ, c) pair.

Now, we can check for the stability of the mixed state at each configuration. If there is only one rest point in the unit interval and $G(1) < 0$ and $G(0) > 0$ the mixed state will be stable. We will use a wider variety of amnesty probabilities: low ($\alpha = 0.1$ and $\alpha = 0.3$), medium ($\alpha = 0.5$) and high ($\alpha = 0.7$ and $\alpha = 0.9$)).

- $v = 2c$, $\epsilon = 4$, $\phi = 0.02$ and $c = 1$: For the first configuration, there are 3 solutions and at most one in the unit interval. $F(0) > 0$ and $F(1) = 0$ for values of α greater than 0.1. In case $\alpha = 0.1$, the one acceptable root will be definitely stable. This means that even though the resource covers the cost of confrontation and there is amnesty, if the probability of amnesty is low enough, it can deter some of the population from acting against the norm.

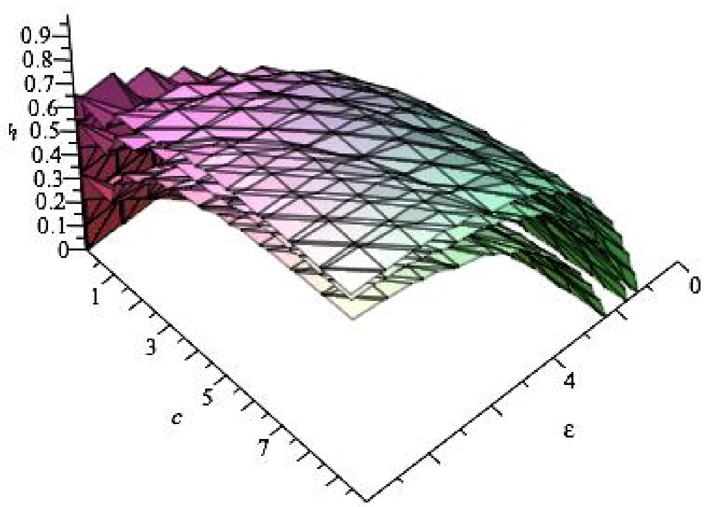


Figure 3.6 – Mixed states for $v = 2c$ and $\phi = 0.02$ for low $\alpha = 0.1$ medium $\alpha = 0.3$ and high $\alpha = 0.5$

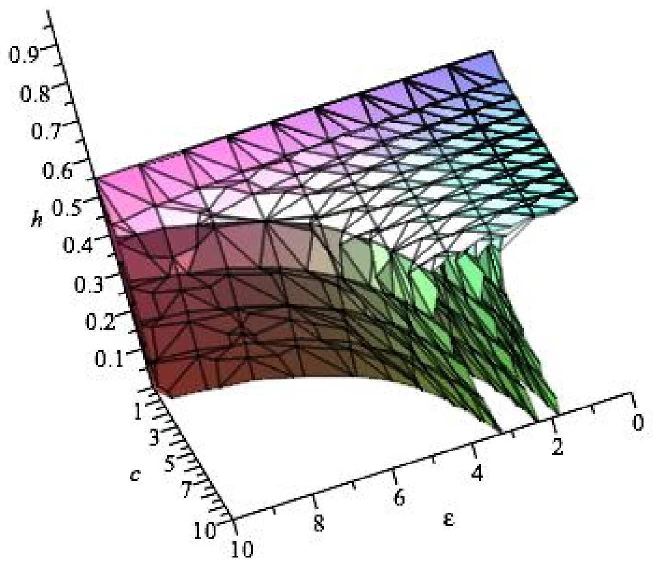


Figure 3.7 – Mixed states for $v = c$ and $\phi = 0.02$ for low $\alpha = 0.1$ medium $\alpha = 0.3$ and high $\alpha = 0.5$

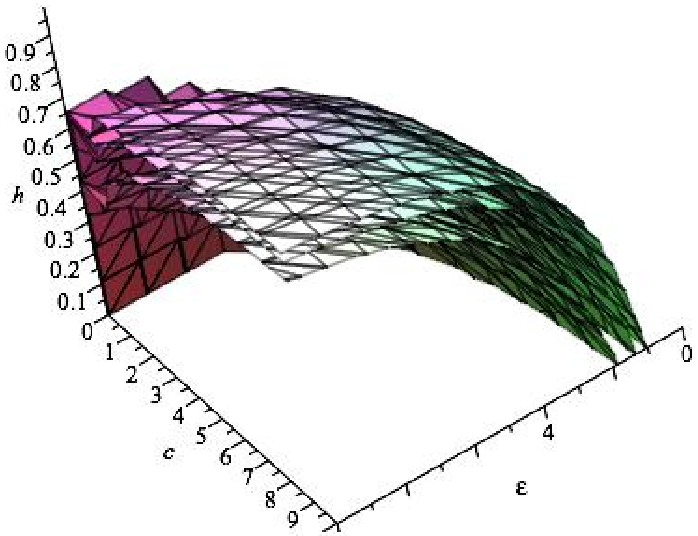


Figure 3.8 – Mixed states for $v = 3c$ and $\phi = 0.02$ for low $\alpha = 0.1$ medium $\alpha = 0.3$ and high $\alpha = 0.5$

- $v = c$, $\epsilon = 4$, $\phi = 0.02$ and $c = 1$: For this configuration, there are 5 solutions and only one in the unit interval. $F(0) > 0$ and $F(1) < 0$. In case the one acceptable root will be definitely stable. We have illustrated this case in Figure 3.10, we see that this rest point is near 0.5 for different values of α from lower to higher ones.

- $v = 3c$, $\epsilon = 4$, $\phi = 0.02$ and $c = 1$: For this configuration, there are 3 solutions. $F(0)$ is positive for higher values of α and $F(1) > 0$. For these values, there are no rest points in the unit interval and only criminals survive. Figure 3.3 shows that for $\alpha = 0.1$ there a rest point in the unit interval. For this rest point, $F(0) < 0$ and $F(1) > 0$, we see that both pure states are stable, a population may be invaded by criminals or innocents. Even though the value of the resource is higher than the cost of confrontation, the existence of a certain probability of amnesty does not allow the population to be all criminals.

We see that the introduction of amnesty increases the likelihood of a pure criminal population or a mixed population to be stable. But even though the resource may be of very high value and there is amnesty, there is a chance for the pure state with all

Figure 3.9 – $G(h)$ for $v = c$, $\epsilon = 4$, $\phi = 0.02$, $c = 1$ and $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$

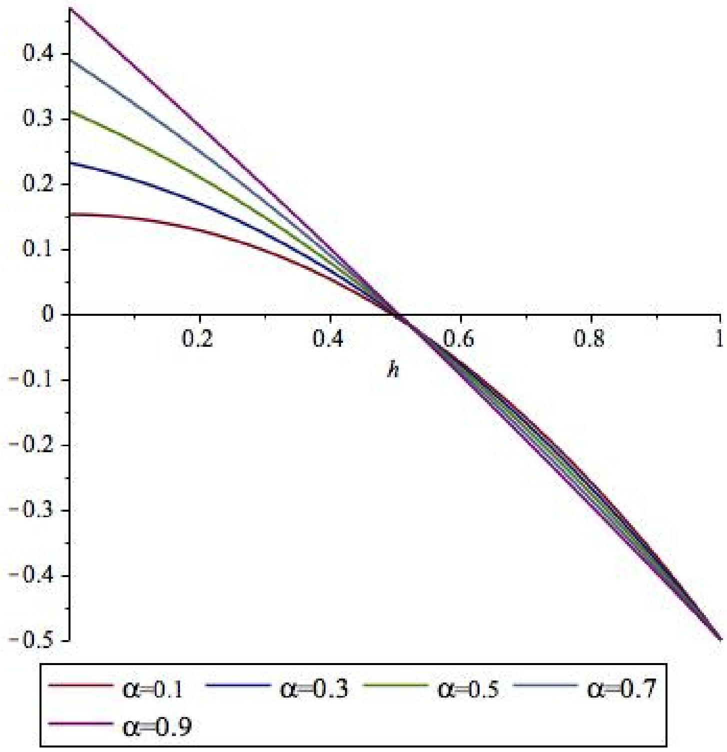
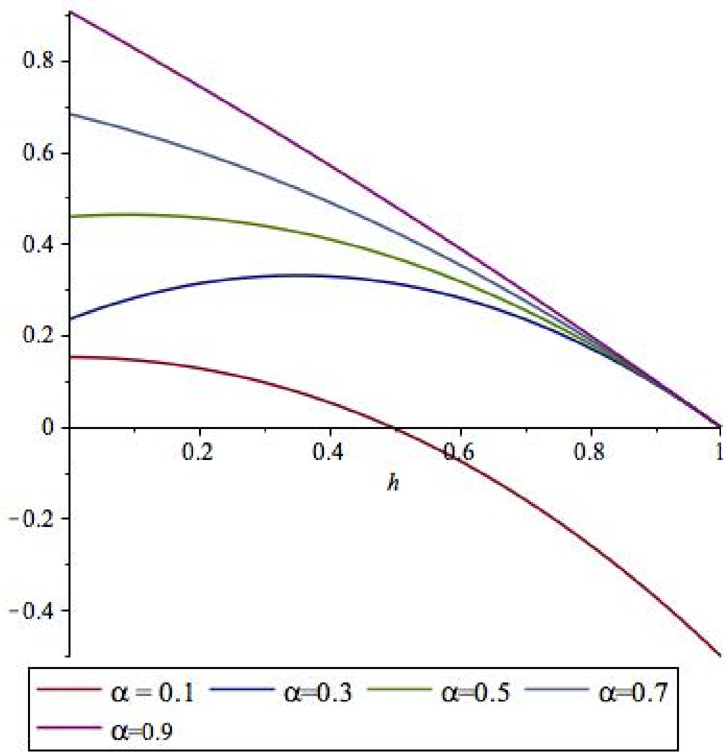
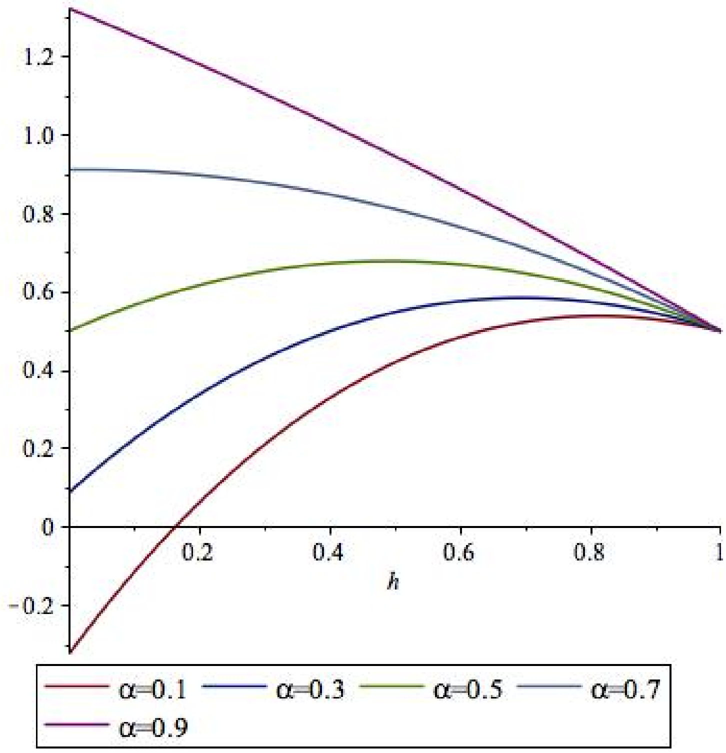


Figure 3.10 – $G(h)$ for $v = 2c$, $\epsilon = 4$, $\phi = 0.02$, $c = 1$ and $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$



innocents to be stable.

Figure 3.11 – $G(h)$ for $v = 3c$, $\epsilon = 4$, $\phi = 0.02$, $c = 1$ and $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$



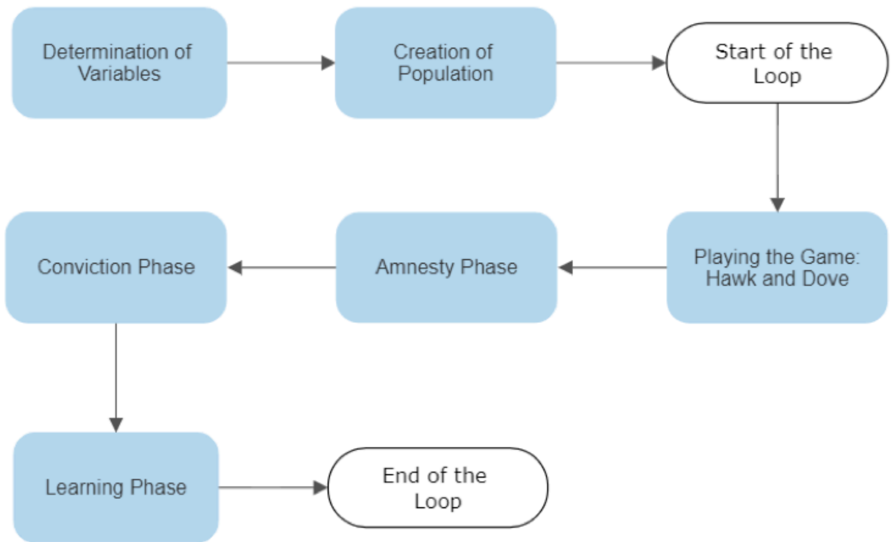
4 AGENT BASED MODEL

In this section we will use simulations to simulate our model over discrete time periods to understand and capture every agent's action precisely. Using a simulation will also allow us to use different variables and change some patterns or stages in our model to test and inspect not only collective behaviors of population, also behavior of an individual agent.

4.1 Description of the simulation

The simulation used in this section consists of various phases, which are sequential to each other and can be seen as occurrence of relevant processes. This will also make our simulation modular, so that we can add or remove phases to test different models. We will loop over some specific phases to make the simulation dynamic. Figure 4.1 illustrates these processes.

Figure 4.1 – Sequential phases of our simulation



In first step, we determine the variables; proportion of hawks over population, number of agents in population, payoff of victory, cost, fee rate, amnesty rate, how many rounds the simulation last, and base payoff. From these variables, our population is created then the loop starts with the game phase. The next process creates a basket of non-convicted agent's ids and shuffles it, then creates pairs for these ids who will play the game (in case of an odd number of non-convicted agents, the last agent automatically takes Dove vs Dove payoff).

After all games are played, there is an amnesty phase, which occurs with a given probability. In this phase all agents status converted to free, but if an agent is in prison it is also recorded that agent granted pardon or amnesty before. Later, from that record we will calculate recidivism rate. The population of individuals has their history of past offenses $k \geq 0$, whether convicted or not. We can partition the population according to their number of past criminal offenses. Then, each time an individual is faced with an opportunity to commit a crime, he decides whether to transition from C_k to C_{k+1}.

Each time, there is a certain proportion of convicted criminals going back to normal (playing the game), going out of prison or being out of conviction. The population of criminals is partitioned according to their history of crimes. There are criminals who can not be granted a remission or exemption and there are criminals who have decided to commit no more crimes.

Next, conviction phase occurs. All agents played Hawk in their last game can be convicted with the detection probability. If an agent is convicted in this phase, their last payoff is lost, and they cannot participate to further games until they are released from conviction. The reason, amnesty phase comes before conviction phase is preventing recently convicted agents to be released before a period passes.

Finally, all agents compare their last payoff with a random agent in all population then decide to change their strategy, if their payoff is less than that agent's payoff, or not, if their payoff greater or equal to this payoff.

4.2 Results of the simulation

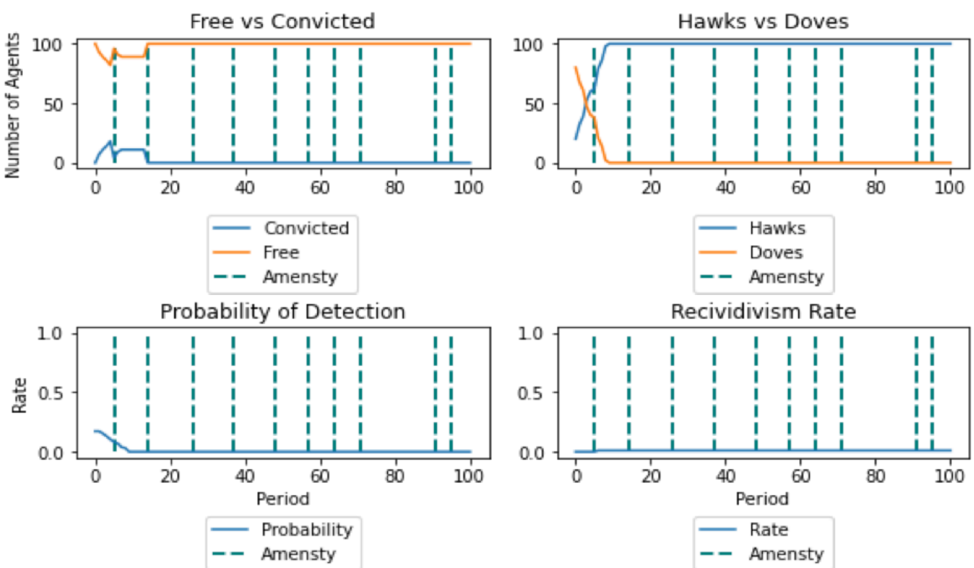
In the previous section's first proposal, we claimed if the value of the resource is high enough i.e. v > 2c, playing against social norm is stable, regardless having a conviction and amnesty mechanic present in the system. For all simulation below we used the same values for the variables and parameters given by Table 4.1 for an easier comparison and illustration.

Population N_0	100
Initial Hawk ratio in the population c_0	20%
Value of the resource v	10
Fee of judicial system ϕ	0.02
Amnesty chance α	0.1
Number of rounds T	100
Efficiency of detection process ϵ	0.1

Table 4.1 – Parameter and variable values used in the simulation

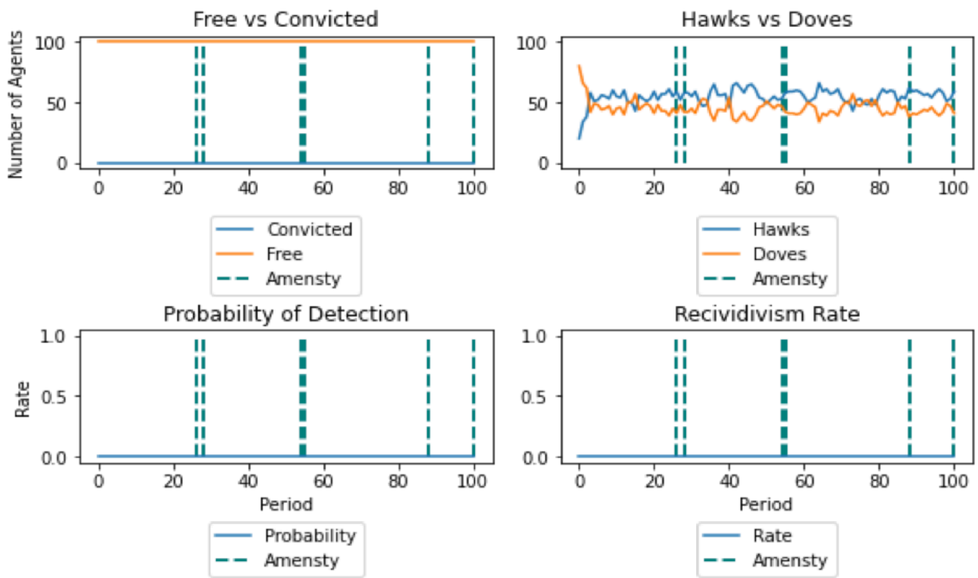
We see that not only, playing Hawk invades the population, but also this reduces further detection probabilities. The recidivism rate here is 0.02, it goes stable after second pardon which happens right after all of the population converts to Hawk (Figure 4.2).

Figure 4.2 – Simulation I ($v > 2c$)



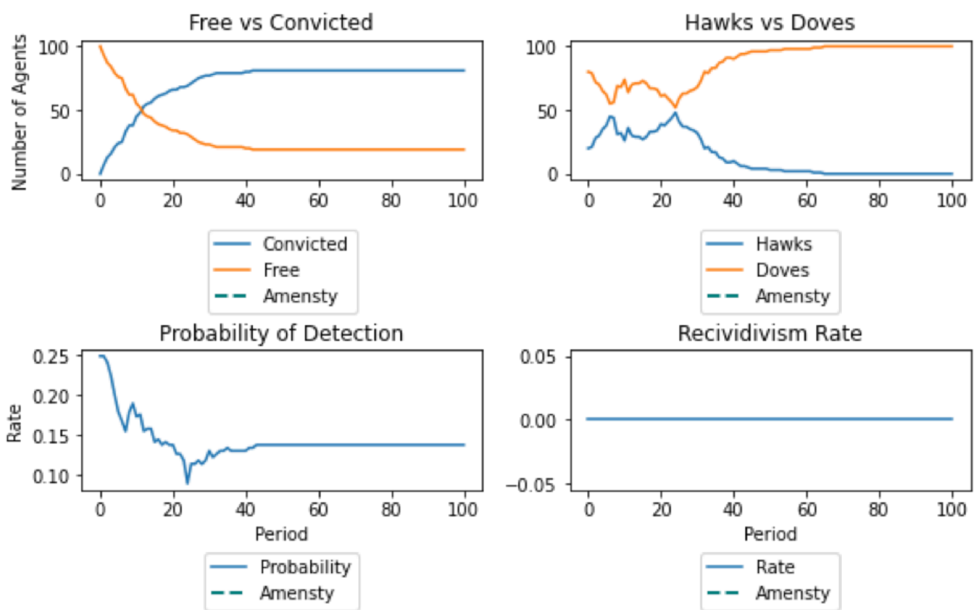
For the scenario of the cost of confrontation equal to the value of the resource, without a conviction and amnesty mechanic, the population converged into half Hawk and half Dove (Figure 4.3).

Figure 4.3 – Simulation II ($v = c$ no imprisonment and no amnesty)



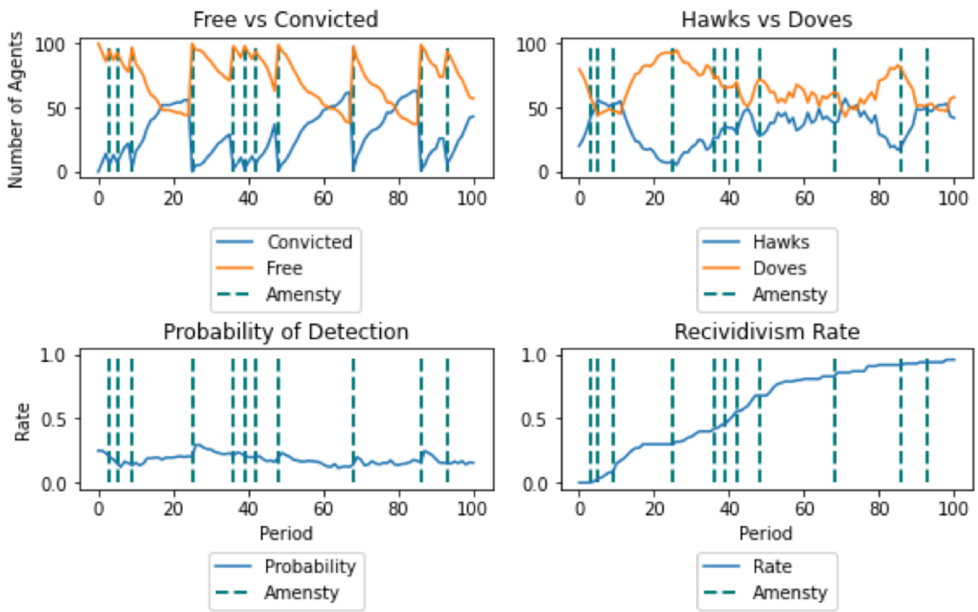
By adding prison mechanics to the picture, we see only Dove population becomes a stable state (Figure 4.4). We should state that, our model does not include reproduction mechanics, therefore the population size stays the same through the simulation.

Figure 4.4 – Simulation III ($v = c$ imprisonment and no amnesty)



And lastly, we introduce amnesty mechanic to the simulation with a probability of 10%. This is in accordance with Turkey's recent history where nearly an amnesty passed in every 10 years in average.

Figure 4.5 – Simulation IV ($v = c$ imprisonment and amnesty)

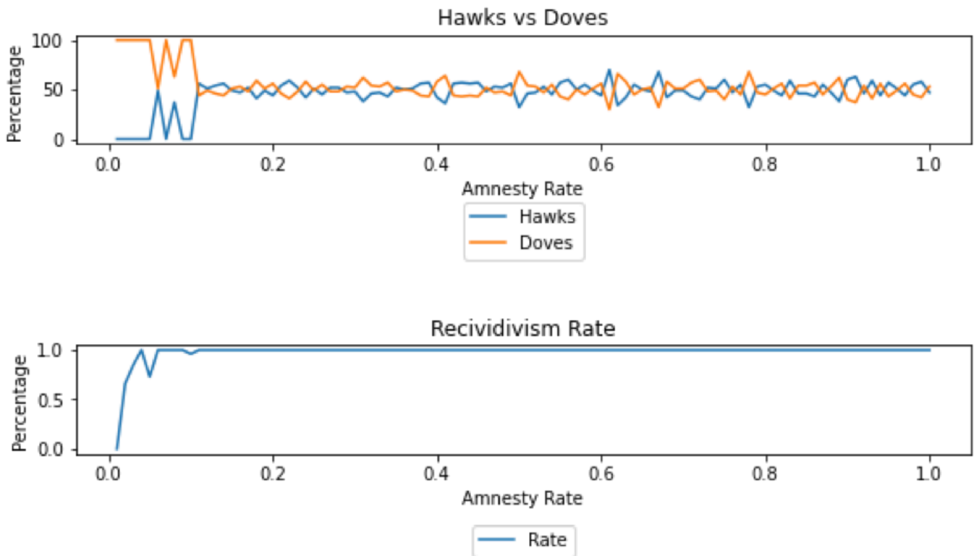


As for results, we can say that not only Doves lose their edge against Hawk players, 87% percentage of population convicted at least 2 times (3.88 times to be precise) and played Hawk 34.86 times in average (Figure 4.5).

4.3 Effect of amnesty on recidivism

In this subsection we will analyze the effect of the chance of amnesty over simulation results. We will change amnesty rates between games and see its affect on recidivism. With increased amnesty rate (over 10%), recidivism rate quickly sits on 100% percentage (Figure 4.6). For population, we see a Dove dominant population quickly changes into a mixed strategy again. Which further proves the point, the amnesty rates increases both crime and recidivism rates across population.

Figure 4.6 – Simulation V



5 CONCLUSION

The subject of this thesis was to understand how sanctions and pre-release policies, amnesty in particular, shapes the population by comparing different steady states between multiple cases using game theory and simulations. Crime and recidivism rates are used to measure this impact, where crime rates denoted by the percentage of players who played against the norms of the society in that period, and recidivism rates calculated in the agent-based model by simply finding agents who were convicted multiple times. The findings of this study are consistent with its main proposal, while sanction mechanisms protecting innocent agents, amnesty not only reduces its effect but also makes more agent criminals. This choice made by agents can be avoided by setting the rules and the laws accordingly.

When an agent of the population decides to commit a crime, in a rational setting where perfect information exists, the agent is aware of the consequences of his actions. It might not be that precise, like an agent may not know how many years it'll serve, or detection probability to its decimal points, but the agent is still willing to pay reference cost in his mind formed by exposure to news, tv, movies, stories, books, et cetera. In the case of a pre-release policy, it works like a discount in a product, suddenly the cost of his actions reduced by half, or vanished completely. When time comes, and the same agent encounters another opportunity of crime, the cost is lower than the first time, where the agent might decide to commit another crime, not because its payoff is strictly better than before, just because the cost is lower. This phenomenon, which can be called "criminal tolerance", also works in populations, where the stories and media connect people. Now another agent tries to form a reference cost in his mind to decide whether to commit a crime or not, this reference cost is also reduced by actions and payoffs of other agents in the population, where eventually every agent is affected by it and the criminal tolerance gets lower every time.

According to the model presented in this study, any chance of amnesty reduces this cost multiplied by detection rate. If an amnesty must be issued, it is assumed to be an ethical one rather than economic or political one, policy makers should increase detection and conviction rates to an extent so that reduction in cost of crime is covered by it. But in case of an economic decision, policy makers must compare the cost of increasing detection rates with the cost of increasing institutional expenditures, and the negative externality imposed on the population if possible. This research attempted to catch this scenario by adding a budget created by fees of innocent players, and it is found agents decide to be criminals rather than paying, as long as another amnesty is coming. And the negative externality created by this situation, should be considered as both monetary and psychologically, similar to previous works. Both being subject of a crime or committing it creates a toll on the agent. Policies aim should be not only protecting monetary resources of government and population, but also protecting the population from psychological tolls. These tolls might generate a negative impact on an agent's monetary wealth, which is another indirect cost.

In future research, network theory or a segmentation over population can be used to understand how amnesty affects subpopulations and total population. This is an important shortcoming of this research and generates higher recidivism rates than expected, due to identical distances between agents. Another important subject is rehabilitation encouraged by policy makers, in this study only rehabilitation probability comes from imitation mechanics. Therefore, the only way to reduce the toll of amnesty is increasing detection rates. But in real life, rehabilitation is an important part of pre-release policies.

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A Simulation

```
class Agent:

  def __init__(self, strategy, fitness, id):
    self._strategy = strategy
    self._id = id
    self._fitness = fitness
    self._history = []
    self._status = 'Free'
    self._statushistory = ['Free']
    self._payoffhistory = []
    self._amnestygranted = 0
    self._convictioncount = 0

  #id property.
  def get_id(self):
    return self._id
  id = property(get_id)

  #strategy property.
  def get_strategy(self):
    return self._strategy
  def set_strategy(self, x):
    self._strategy = x
  strategy = property(get_strategy, set_strategy)

  #fitness property.
  def get_fitness(self):
    return self._fitness
  def set_fitness(self, x):
    self._fitness = x
  fitness = property(get_fitness, set_fitness)
```

```
    #strategy history property.
    def get_history(self):
      return self._history
    def set_history(self, x):
      self._history.append(x)
    history = property(get_history, set_history)

    #status property.
    def get_status(self):
      return self._status
    def set_status(self, x):
      self._status = x
    status = property(get_status, set_status)

    #status history.
    def get_statushistory(self):
      return self._statushistory
    def set_statushistory(self, x):
      self._statushistory.append(x)
    statushistory = property(get_statushistory, set_statushistory)

    #payoff history.
    def get_payoffhistory(self):
      return self._payoffhistory
    def set_payoffhistory(self, x):
      self._payoffhistory.append(x)
    payoffhistory = property(get_payoffhistory, set_payoffhistory)

    #amnesty granted.
    def get_amnestygranted(self):
      return self._amnestygranted
    def set_amnestygranted(self, x):
```

```
    self._amnestygranted = x
  amnestygranted = property(get_amnestygranted, set_amnestygranted)

  #convictioncount.
  def get_convictioncount(self):
    return self._convictioncount
  def set_convictioncount(self, x):
    self._convictioncount = x
  convictioncount = property(get_convictioncount, set_convictioncount)

def create_population(n, proportion):
  if isinstance(n * proportion, int):
    raise Exception('Created population does not yield an integer')
  elif proportion >= 1:
    raise Exception('Proportion should not be equal or greated than 1.')
  else:
    _first = int(n * proportion)
    _last = int(n - _first)
    _population = []
    for i in range(_first):
      _population.append(Agent('Hawk', 0, i))
    for i in range(_last):
      _population.append(Agent('Dove', 0, _first+i))
    return _population

def hawk_and_dove(agent0, agent1, v, c, b, f):
  if agent0.strategy == 'Hawk' and agent1.strategy == 'Hawk':
    agent0.fitness += b+(v/2)-c
    agent1.fitness += b+(v/2)-c
```

```
    agent0.payoffhistory = b+(v/2)-c
    agent1.payoffhistory = b+(v/2)-c
  elif agent0.strategy == 'Dove' and agent1.strategy == 'Hawk':
    agent0.fitness += b*(1-f)
    agent1.fitness += v+b
    agent0.payoffhistory = b*(1-f)
    agent1.payoffhistory = v+b
  elif agent0.strategy == 'Hawk' and agent1.strategy == 'Dove':
    agent0.fitness += v+b
    agent1.fitness += b*(1-f)
    agent0.payoffhistory = v+b
    agent1.payoffhistory = b*(1-f)
  else:
    agent0.fitness += (b+v/2)*(1-f)
    agent1.fitness += (b+v/2)*(1-f)
    agent0.payoffhistory = (b+v/2)*(1-f)
    agent1.payoffhistory = (b+v/2)*(1-f)

def conviction_phase(agent, p):
  _random = random.randrange(100)
  _p = (p*100)-1
  if agent.strategy == 'Hawk':
    if _random < _p:
      agent.status = 'Convicted'
      agent.fitness -= agent.payoffhistory[-1]
      agent.payoffhistory[-1] = 0
      agent.convictioncount += 1
    else:
      pass

def learning_phase(agent, population):
  agent.history = agent.strategy
  _random = random.randrange(len(population))
```

```python
  if agent.payoffhistory[-1] >= population[_random].payoffhistory[-1]:
    pass
  else:
    agent.strategy = population[_random].strategy

def calculate_budget(v, n, f, b, e):
  _t = 0
  for i in range(len(n)):
    if n[i].strategy == 'Dove':
      _t += n[i].payoffhistory[-1]*f/(1-f)
  _e = e
  return _e*sqrt(_t)

def amnesty_phase(n, q):
  for i in range(len(n)):
    n[i].statushistory = n[i].status
  _random = random.randrange(100)
  _q = (q*100)-1
  if _random < _q:
    for i in range(len(n)):
      if n[i].statushistory[-1] == 'Convicted':
        n[i].amnestygranted = 1
        n[i].status = 'Free'
      else:
        n[i].status = 'Free'
    else:
      pass
    return 1
  else:
    return 0

import random
```

```
import sympy as sy
from math import sqrt

#değişkenler.
number = 100
proportion = 0.2
n = create_population(number, proportion)
#victory.
v = 10
#cost.
c = 21
#fee rate.
f = 0.02
#amnesty rate.
q = 0.1
#kaç round.
r = 100
#base puan.
b = 0
#e.
e = 0.1

#output listeleri.
nh = [number*proportion]
nd = [number*(1-proportion)]
nc = [0]
nf = [number]
ar = [0]
pr = []
rr = [0]

for j in range(r):
  #sabit conviction rate için buraya başvurulabilir.
```

```
#serbest popülasyonun id numaralarından oluşan bir sepetin
↪ oluşturulması ve sepetin karıştırılması. (else, base pay
↪ off'un convicted agenta verilmesi)
fn = []
for i in range(len(n)):
  if n[i].status == 'Free':
    fn.append(n[i].id)
  else:
    n[i].payoffhistory = 0
  random.shuffle(fn)
#pairwise olarak agentlara hawk and dove oynatılması.
for i in range(0, len(fn), 2):
  try:
    hawk_and_dove(n[fn[i]], n[fn[i+1]], v, c, b, f)
  except:
    n[fn[i]].fitness += (v/2)*(1-f)
    n[fn[i]].payoffhistory = (v/2)*(1-f)
#sabit conviction rate için buraya başvurulabilir.
#calculate budget.
p = calculate_budget(v, n, f, b, e)
#amnesty fazı.
bin_amnesty = amnesty_phase(n, q)
#tutuklanma fazı.
for i in range(0, len(fn)):
  conviction_phase(n[fn[i]], p)
#öğrenme fazı.
for i in range(0, len(n)):
  learning_phase(n[i], n)
#popülasyon dağılımı.
n_hawks = 0
n_doves = 0
n_convicted = 0
n_free = 0
```

```
n_recidivism = 0
for i in range(len(n)):
  if n[i].status == 'Free':
    n_free += 1
  else:
    n_convicted += 1
for i in range(len(n)):
  if n[i].strategy == 'Hawk':
    n_hawks += 1
  else:
    n_doves += 1
for i in range(len(n)):
  if n[i].convictioncount > 1:
    n_recidivism += 1
try:
  n_recidivism = n_recidivism/len(n)
except:
  n_recidivism = 0
print('---')
print(f'Round = {j+1}')
print(f'Did an Amnesty Issued = {bin_amnesty}')
ar.append(bin_amnesty)
print(f'Number of Hawks = {n_hawks}')
nh.append(n_hawks)
print(f'Number of Doves = {n_doves}')
nd.append(n_doves)
print(f'Number of Free = {n_free}')
nf.append(n_free)
print(f'Number of Convicted = {n_convicted}')
nc.append(n_convicted)
print(f'Probability of Conviction = {p}')
pr.append(p)
print(f'Recidivism Rate = {n_recidivism}')
```

```
  rr.append(n_recidivism)
  print('---')

import matplotlib.pyplot as plt

#2x2 tabloların oluşturulması.
fig, spl = plt.subplots(2,2, figsize = (8,5))

x = [i+1 for i in range(r)]

#sadece pr'a ilk element ekeleme (100 element olması için).
pr = [pr[0]] + pr

ar_v = []
for i in range(len(ar)):
  if ar[i] == 1:
    ar_v.append(i)

#plot of population.
spl[0,0].set_title('Free vs Convicted')
#spl[0,0].xlabel('Period')
#spl[0,0].ylabel('Number of Agents')
spl[0,0].plot(nc, label = 'Convicted')
spl[0,0].plot(nf, label = 'Free')
spl[0,0].vlines([ar_v], ymin = [0], ymax = [100], colors='teal',
↪  ls='--', lw=2, label='Amensty')
spl[0,0].legend(loc='center', bbox_to_anchor=(0.25, -0.9, 0.5,
↪  0.5))

#plot of population.
spl[0,1].set_title('Hawks vs Doves')
#spl[0,1].xlabel('Period')
#spl[0,1].ylabel('Number of Agents')
```

```
spl[0,1].plot(nh, label = 'Hawks')
spl[0,1].plot(nd, label = 'Doves')
spl[0,1].vlines([ar_v], ymin = [0], ymax = [100], colors='teal',
↪  ls='--', lw=2, label='Amensty')
spl[0,1].legend(loc='center', bbox_to_anchor=(0.25, -0.9, 0.5,
↪  0.5))

#propability of apprehension.
spl[1,0].set_title('Probability of Detection')
#spl[1,0].xlabel('Period')
#spl[1,0].ylabel('Probability')
spl[1,0].plot(pr, label = 'Probability')
spl[1,0].vlines([ar_v], ymin = [0], ymax = [1], colors='teal',
↪  ls='--', lw=2, label='Amensty')
spl[1,0].legend(loc='center', bbox_to_anchor=(0.25, -0.9, 0.5,
↪  0.5))

#propability of apprehension.
spl[1,1].set_title('Recividivism Rate')
#spl[1,1].xlabel('Period')
#spl[1,1].ylabel('Rate')
spl[1,1].plot(rr, label = 'Rate')
spl[1,1].vlines([ar_v], ymin = [0], ymax = [1], colors='teal',
↪  ls='--', lw=2, label='Amensty')
spl[1,1].legend(loc='center', bbox_to_anchor=(0.25, -0.9, 0.5,
↪  0.5))

plt.subplots_adjust(left=0.1,
                    bottom=1.5,
                    right=0.9,
                    top=2,
                    wspace=1,
                    hspace=0.5)
```

```
plt.setp(spl[-1, :], xlabel='Period')
plt.setp(spl[:2,0], ylabel='Rate')
plt.setp(spl[:1,0], ylabel='Number of Agents')
plt.tight_layout()
```